Fluid Mechanics

Fluid is the name given to a substance which begins to flow when external force is applied on it. Liquids and gases are fluids. Fluids do not have their own shape but take the shape of the containing vessel. The branch of physics which deals with the study of fluids at rest is called hydrostatics and the branch which deals with the study of fluids in motion is called hydrodynamics.

11.1 Pressure

The normal force exerted by liquid at rest on a given surface in contact with it is called thrust of liquid on that surface.

The normal force (or thrust) exerted by liquid at rest per unit area of the surface in contact with it is called pressure of liquid or hydrostatic pressure.

If F be the normal force acting on a surface of area A in contact with liquid, then pressure exerted by liquid on this surface is $P = F/A$

(1) Units : N/m^2 or Pascal (S.I.) and Dyne/cm^2 (C.G.S.)

(2) Dimension : $[P] = \frac{[F]}{[A]} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$

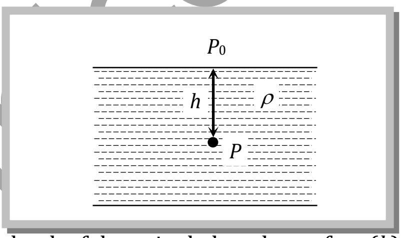
(3) At a point pressure acts in all directions and a definite direction is not associated with it. So pressure is a tensor quantity.

(4) Atmospheric pressure : The gaseous envelope surrounding the earth is called the earth's atmosphere and the pressure exerted by the atmosphere is called atmospheric pressure. Its value on the surface of the earth at sea level is nearly $1.013 \times 10^5 \, N/m^2$ or Pascal in S.I. other practical units of pressure are atmosphere, bar and torr (*mm* of *Hg*)

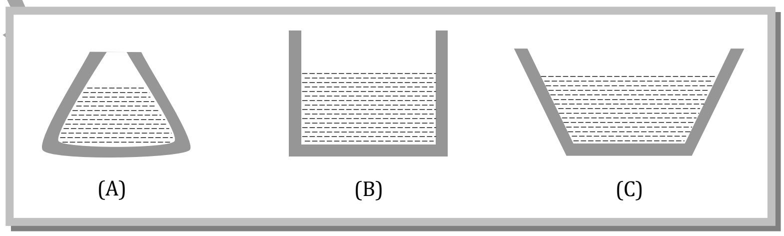
$$1atm = 1.01 \times 10^5 \, Pa = 1.01 \, \text{bar} = 760 \ \text{torr}$$

The atmospheric pressure is maximum at the surface of earth and goes on decreasing as we move up into the earth's atmosphere.

(5) If P_0 is the atmospheric pressure then for a point at depth h below the surface of a liquid of density ρ, hydrostatic pressure P is given by $P = P_0 + h\rho g$

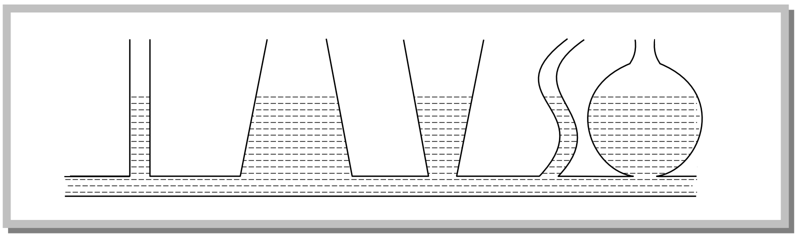


(6) Hydrostatic pressure depends on the depth of the point below the surface (h), nature of liquid (ρ) and acceleration due to gravity (g) while it is independent of the amount of liquid, shape of the container or cross-sectional area considered. So if a given liquid is filled in vessels of different shapes to same height, the pressure at the base in each vessel's will be the same, though the volume or weight of the liquid in different vessels will be different.



$P_A = P_B = P_C$ but $W_A < W_B < W_C$

(7) In a liquid at same level, the pressure will be same at all points, if not, due to pressure difference the liquid cannot be at rest. This is why the height of liquid is the same in vessels of different shapes containing different amounts of the same liquid at rest when they are in communication with each other.



(8) Gauge pressure : The pressure difference between hydrostatic pressure P and atmospheric pressure P_0 is called gauge pressure. $P - P_0 = h\rho g$

Sample problems based on Pressure

Problem 1. If pressure at half the depth of a lake is equal to 2/3 pressure at the bottom of the lake then what is the depth of the lake **[RPET 2000]**

(a) 10 *m* (b) 20 *m* (c) 60 *m* (d) 30 *m*

Solution : (b) Pressure at bottom of the lake = $P_0 + h\rho g$ and pressure ay half the depth of a lake $= P_0 + \frac{h}{2}\rho g$

According to given condition $P_0 + \frac{1}{2}h\rho g = \frac{2}{3}(P_0 + h\rho g) \Rightarrow \frac{1}{3}P_0 = \frac{1}{6}h\rho g \Rightarrow h = \frac{2P_0}{\rho g} = \frac{2\times 10^5}{10^3 \times 10} = 20\,m$.

Problem 2. Two bodies are in equilibrium when suspended in water from the arms of a balance. The mass of one body is 36 *g* and its density is 9 g/cm^3. If the mass of the other is 48 *g*, its density in g/cm^3 is **[CBSE 1994]**

(a) $\frac{4}{3}$ (b) $\frac{3}{2}$ (c) 3 (d) 5

Solution : (c) Apparent weight $= V(\rho - \sigma)g = \frac{m}{\rho}(\rho - \sigma)g$

where $m =$ mass of the body, $\rho =$ density of the body and $\sigma =$ density of water

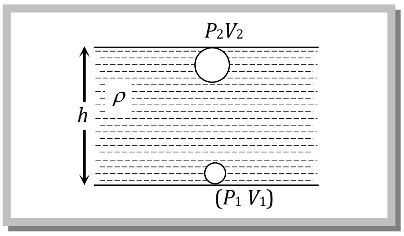
If two bodies are in equilibrium then their apparent weight must be equal.

$\therefore \frac{m_1}{\rho_1}(\rho_1 - \sigma)g = \frac{m_2}{\rho_2}(\rho_2 - \sigma)g \Rightarrow \frac{36}{9}(9-1) = \frac{48}{\rho_2}(\rho_2 - 1)g$. By solving we get $\rho_2 = 3$.

Problem 3. An inverted bell lying at the bottom of a lake 47.6 *m* deep has 50 cm^3 of air trapped in it. The bell is brought to the surface of the lake. The volume of the trapped air will be (atmospheric pressure = 70 *cm* of *Hg* and density of *Hg* = 13.6 g/cm^3) **[CPMT 1989]**

(a) 350 cm^3 (b) 300 cm^3 (c) 250 cm^3 (d) 22 cm^3

Solution : (b) According to Boyle's law, pressure and volume are inversely proportional to each other *i.e.* $P \propto \frac{1}{V}$



$\Rightarrow P_1V_1 = P_2V_2$

$\Rightarrow (P_0 + h\rho_w g)V_1 = P_0V_2$

$\Rightarrow V_2 = \left(1 + \frac{h\rho_w g}{P_0}\right)V_1$

$\Rightarrow V_2 = \left(1 + \frac{47.6 \times 10^2 \times 1 \times 1000}{70 \times 13.6 \times 1000}\right)V_1$ [As $P_2 = P_0 = 70\ cm$ of $Hg = 70 \times 13.6 \times 1000$]

$\Rightarrow V_2 = (1+5)50\,cm^3 = 300\,cm^3$.

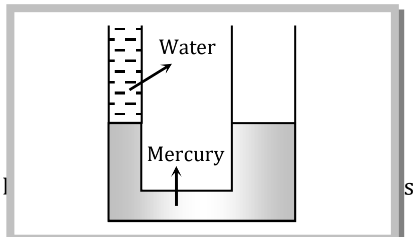
Problem 4. A U-tube in which the cross-sectional area of the limb on the left is one quarter, the limb on the right contains mercury (density 13.6 g/cm^3). The level of mercury in the narrow limb is at a distance of 36 *cm* from the upper end of the tube. What will be the rise in the level of mercury in the right limb if the left limb is filled to the top with water

(a) 1.2 *cm*

(b) 2.35 *cm*

(c) 0.56 *cm*

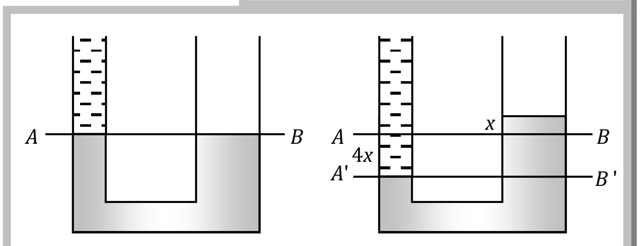
(d) 0.8 *cm*



Solution : (c) If the rise of level in the right limb be *x cm.* the fall of level of mercury in left l… s section of right limb 4 times as that of left limb.

$\therefore$ Level of water in left limb is (36 + 4*x*) *cm*.

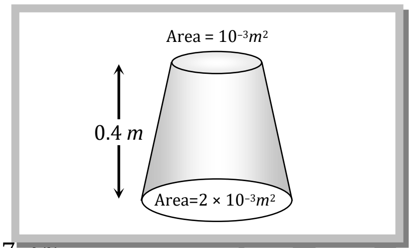
Now equating pressure at interface of mercury and water (at $A'B'$)



$(36 + 4x) \times 1 \times g = 5x \times 13.6 \times g$

By solving we get $x = 0.56$ *cm.*

***Problem* 5.** A uniformly tapering vessel is filled with a liquid of density 900 kg/m^3. The force that acts on the base of the vessel due to the liquid is $(g = 10\, ms^{-2})$



(a) 3.6 *N*

(b) 7.2 *N*

(c) 9.0 *N*

(d) 14.4 *N*

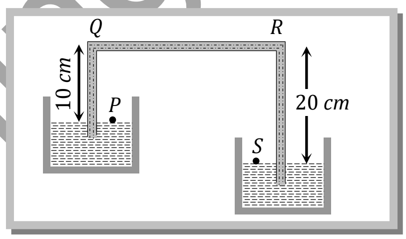
Solution : (b) Force acting on the base $F = P \times A = hdgA = 0.4 \times 900 \times 10 \times 2 \times 10^{-3} = 7.2 N$

***Problem* 6.** A tank 5 *m* high is half filled with water and then is filled to the top with oil of density 0.85 g/cm^3. The pressure at the bottom of the tank, due to these liquids is

(a) 1.85 g/cm^2 (b) 89.25 g/cm^2 (c) 462.5 g/cm^2 (d) 500 g/cm^2

Solution : (c) Pressure at the bottom $P = (h_1 d_1 + h_2 d_2)\frac{g}{cm^2} = [250 \times 1 + 250 \times 0.85] = 250\ [1.85]\frac{g}{cm^2} = 462.5\frac{g}{cm^2}$

***Problem* 7.** A siphon in use is demonstrated in the following figure. The density of the liquid flowing in siphon is 1.5 *gm/cc*. The pressure difference between the point *P* and *S* will be



(a) 10^5 *N/m*

(b) 2×10^5 *N/m*

(c) Zero

(d) Infinity

Solution : (c) As the both points are at the surface of liquid and these points are in the open atmosphere. So both point possess similar pressure and equal to 1 *atm.* Hence the pressure difference will be zero.

***Problem* 8.** The height of a mercury barometer is 75 *cm* at sea level and 50 *cm* at the top of a hill. Ratio of density of mercury to that of air is 10^4. The height of the hill is

(a) 250 *m* (b) 2.5 *km* (c) 1.25 *km* (d) 750 *m*

Solution : (b) Difference of pressure between sea level and the top of hill

$$\Delta P = (h_1 - h_2) \times \rho_{Hg} \times g = (75 - 50) \times 10^{-2} \times \rho_{Hg} \times g \quad ......(i)$$

and pressure difference due to *h* meter of air $\Delta P = h \times \rho_{air} \times g$ ......(ii)

By equating (i) and (ii) we get $h \times \rho_{air} \times g = (75 - 50) \times 10^{-2} \times \rho_{Hg} \times g$

$\therefore h = 25 \times 10^{-2}\left(\frac{\rho_{Hg}}{\rho_{air}}\right) = 25 \times 10^{-2} \times 10^4 = 2500\ m$ $\therefore$ Height of the hill = 2.5 *km.*

11.2 Density

In a fluid, at a point, density ρ is defined as: $\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} = \frac{dm}{dV}$

(1) In case of homogenous isotropic substance, it has no directional properties, so is a scalar.

(2) It has dimensions $[ML^{-3}]$ and S.I. unit kg/m^3 while C.G.S. unit *g/cc* with $1 g/cc = 10^3 kg/m^3$

(3) Density of substance means the ratio of mass of substance to the volume occupied by the substance while density of a body means the ratio of mass of a body to the volume of the body. So for a solid body.

Density of body = Density of substance

While for a hollow body, density of body is lesser than that of substance $[\text{As } V_{\text{body}} > V_{\text{sub.}}]$

(4) When immiscible liquids of different densities are poured in a container the liquid of highest density will be at the bottom while that of lowest density at the top and interfaces will be plane.

(5) Sometimes instead of density we use the term relative density or specific gravity which is defined as :

$$RD = \frac{\text{Density of body}}{\text{Density of water}}$$

(6) If m_1 mass of liquid of density ρ_1 and m_2 mass of density ρ_2 are mixed, then as

$$m = m_1 + m_2 \text{ and } V = (m_1/\rho_1) + (m_2/\rho_2) \qquad [\text{As } V = m/\rho]$$

$$\rho = \frac{m}{V} = \frac{m_1 + m_2}{(m_1/\rho_1) + (m_2/\rho_2)} = \frac{\sum m_i}{\sum(m_i/p_i)}$$

If $m_1 = m_2$ $\quad \rho = \dfrac{2\rho_1\rho_2}{\rho_1 + \rho_2}$ = Harmonic mean

(7) If V_1 volume of liquid of density ρ_1 and V_2 volume of liquid of density ρ_2 are mixed, then as:

$$m = \rho_1 V_1 + \rho_2 V_2 \text{ and } V = V_1 + V_2 \qquad [\text{As } \rho = m/V]$$

If $V_1 = V_2 = V$ $\quad \rho = (\rho_1 + \rho_2)/2$ = Arithmetic Mean

(8) With rise in temperature due to thermal expansion of a given body, volume will increase while mass will remain unchanged, so density will decrease, *i.e.*,

$$\frac{\rho}{\rho_0} = \frac{(m/V)}{(m/V_0)} = \frac{V_0}{V} = \frac{V_0}{V_0(1+\gamma\Delta\theta)} \qquad [\text{As } V = V_0(1+\gamma\Delta\theta)]$$

or $\quad \rho = \dfrac{\rho_0}{(1+\gamma\Delta\theta)} \simeq \rho_0(1-\gamma\Delta\theta)$

(9) With increase in pressure due to decrease in volume, density will increase, *i.e.*,

$$\frac{\rho}{\rho_0} = \frac{(m/V)}{(m/V_0)} = \frac{V_0}{V} \qquad [\text{As } \rho = \frac{m}{V}]$$

But as by definition of bulk-modulus

$$B = -V_0 \frac{\Delta p}{\Delta V} \text{ i.e., } V = V_0\left[1 - \frac{\Delta p}{B}\right]$$

So $\quad \rho = \rho_0\left(1 - \dfrac{\Delta p}{B}\right)^{-1} \simeq \rho_0\left(1 + \dfrac{\Delta p}{B}\right)$

Sample problems based on Density

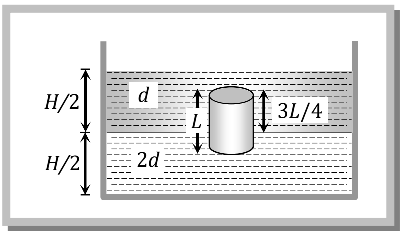
<u>*Problem*</u> 9. A homogeneous solid cylinder of length $L(L < H/2)$. Cross-sectional area $A/5$ is immersed such that it floats with its axis vertical at the liquid-liquid interface with length $L/4$ in the denser liquid as shown in the fig. The lower density liquid is open to atmosphere having pressure P_0. Then density *D* of solid is given by **[IIT-JEE1995]**

(a) $\frac{5}{4}d$

(b) $\frac{4}{5}d$

(c) Ad

(d) $\frac{d}{5}$



Solution : (a) Weight of cylinder = upthrust due to both liquids

$$V \times D \times g = \left(\frac{A}{5}.\frac{3}{4}L\right) \times d \times g + \left(\frac{A}{5}.\frac{L}{4}\right) \times 2d \times g \Rightarrow \left(\frac{A}{5}.L\right)D.g = \frac{A\,L\,d\,g}{4} \Rightarrow \frac{D}{5} = \frac{d}{4} \quad \therefore D = \frac{5}{4}d$$

<u>*Problem*</u> 10. Density of ice is ρ and that of water is σ. What will be the decrease in volume when a mass *M* of ice melts

(a) $\dfrac{M}{\sigma - \rho}$ (b) $\dfrac{\sigma - \rho}{M}$ (c) $M\left[\dfrac{1}{\rho} - \dfrac{1}{\sigma}\right]$ (d) $\dfrac{1}{M}\left[\dfrac{1}{\rho} - \dfrac{1}{\sigma}\right]$

Solution : (c) Volume of ice $= \frac{M}{\rho}$, volume of water $= \frac{M}{\sigma}$ $\therefore$ Change in volume $= \frac{M}{\rho} - \frac{M}{\sigma} = M\left(\frac{1}{\rho} - \frac{1}{\sigma}\right)$

***Problem* 11.** Equal masses of water and a liquid of density 2 are mixed together, then the mixture has a density of

(a) 2/3 (b) 4/3 (c) 3/2 (d) 3

Solution : (b) If two liquid of equal masses and different densities are mixed together then density of mixture

$$\rho = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2} = \frac{2 \times 1 \times 2}{1+2} = \frac{4}{3}$$

***Problem* 12.** Two substances of densities ρ_1 and ρ_2 are mixed in equal volume and the relative density of mixture is 4. When they are mixed in equal masses, the relative density of the mixture is 3. The values of ρ_l and ρ_2 are

(a) $\rho_1 = 6$ and $\rho_2 = 2$ (b) $\rho_1 = 3$ and $\rho_2 = 5$ (c) $\rho_1 = 12$ and $\rho_2 = 4$ (d) None of these

Solution : (a) When substances are mixed in equal volume then density $= \frac{\rho_1 + \rho_2}{2} = 4 \Rightarrow \rho_1 + \rho_2 = 8$ ....(i)

When substances are mixed in equal masses then density $= \frac{2\rho_1\rho_2}{\rho_1 + \rho_2} = 3 \Rightarrow 2\rho_1\rho_2 = 3(\rho_1 + \rho_2)$ ....(ii)

By solving (i) and (ii) we get $\rho_1 = 6$ and $\rho_2 = 2$.

***Problem* 13.** A body of density d_1 is counterpoised by Mg of weights of density d_2 in air of density *d*. Then the true mass of the body is

(a) M (b) $M\left(1 - \frac{d}{d_2}\right)$ (c) $M\left(1 - \frac{d}{d_1}\right)$ (d) $\frac{M(1 - d/d_2)}{(1 - d/d_1)}$

Solution : (d) Let M_0 = mass of body in vacuum.

Apparent weight of the body in air = Apparent weight of standard weights in air

$\Rightarrow$ Actual weight – upthrust due to displaced air = Actual weight – upthrust due to displaced air

$$\Rightarrow M_0 g - \left(\frac{M_0}{d_1}\right)dg = Mg - \left(\frac{M}{d_2}\right)dg \Rightarrow M_0 = \frac{M\left[1 - \frac{d}{d_2}\right]}{\left[1 - \frac{d}{d_1}\right]}.$$

11.3 Pascal's Law

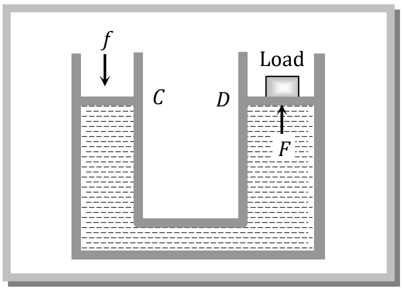
It states that if gravity effect is neglected, the pressure at every point of liquid in equilibrium of rest is same.

or

The increase in pressure at one point of the enclosed liquid in equilibrium of rest is transmitted equally to all other points of the liquid and also to the walls of the container, provided the effect of gravity is neglected.

Example : Hydraulic lift, hydraulic press and hydraulic brakes

Working of hydraulic lift : It is used to lift the heavy loads. If a small force *f* is applied on piston of *C* then the pressure exerted on the liquid



$P = f / a$ [*a* = Area of cross section of the piston in *C*]

This pressure is transmitted equally to piston of cylinder *D*.

Hence the upward force acting on piston of cylinder *D*.

$$F = PA = \frac{f}{a}A = f\left(\frac{A}{a}\right)$$

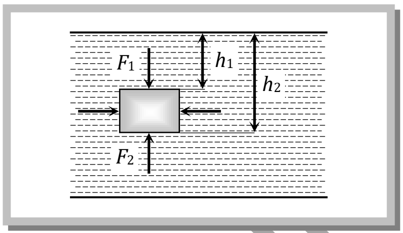
As $A >> a$, therefore $F >> f$. So heavy load placed on the larger piston is easily lifted upwards by applying a small force.

11.4 Archimedes Principle

Accidentally Archimedes discovered that when a body is immersed partly or wholly in a fluid, in rest it is buoyed up with a force equal to the weight of the fluid displaced by the body. This principle is called Archimedes principle and is a necessary consequence of the laws of fluid statics.

When a body is partly or wholly dipped in a fluid, the fluid exerts force on the body due to hydrostatic pressure. At any small portion of the surface of the body, the force exerted by the fluid is perpendicular to the surface and is equal to the pressure at that point multiplied by the area. The resultant of all these constant forces is called upthrust or buoyancy.

To determine the magnitude and direction of this force consider a body immersed in a fluid of density σ as shown in fig. The forces on the vertical sides of the body will cancel each other. The top surface of the body will experience a downward force.



$F_1 = AP_1 = A(h_1 \sigma g + P_0)$ [As $P = h\sigma g + P_0$]

While the lower face of the body will experience an upward force.

$F_2 = AP_2 = A(h_2 \sigma g + P_0)$

As $h_2 > h_1, F_2$ will be greater than F_1, so the body will experience a net upward force

$F = F_2 - F_1 = A\sigma g(h_2 - h_1)$

If *L* is the vertical height of the body $F = A\sigma gL = V\sigma g$ [As $V = AL = A(h_2 - h_1)$]]

i.e., *F* = Weight of fluid displaced by the body.

This force is called upthrust or buoyancy and acts vertically upwards (opposite to the weight of the body) through the centre of gravity of displaced fluid (called centre of buoyancy). Though we have derived this result for a body fully submerged in a fluid, it can be shown to hold good for partly submerged bodies or a body in more than one fluid also.

(1) Upthrust is independent of all factors of the body such as its mass, size, density etc. except the volume of the body inside the fluid.

(2) Upthrust depends upon the nature of displaced fluid. This is why upthrust on a fully submerged body is more in sea water than in fresh water because its density is more than fresh water.

(3) Apparent weight of the body of density (ρ) when immersed in a liquid of density (σ).

Apparent weight = Actual weight – Upthrust $= W - F_{up} = V\rho g - V\sigma g = V(\rho - \sigma)g = V\rho g\left(1 - \frac{\sigma}{\rho}\right)$

$\therefore \quad W_{APP} = W\left(1 - \frac{\sigma}{\rho}\right)$

(4) If a body of volume *V* is immersed in a liquid of density σ then its weight reduces.

W_1 = Weight of the body in air, W_2 = Weight of the body in water

Then apparent (loss of weight) $W_1 - W_2 = V\sigma g \quad \therefore \quad V = \frac{W_1 - W_2}{\sigma g}$

(5) Relative density of a body (R.D.)= $\frac{\text{density of body}}{\text{density of water}} = \frac{\text{Weight of body}}{\text{Weight of equal volu me of water}} = \frac{\text{Weight of body}}{\text{Water thru st}}$

$= \frac{\text{Weight of body}}{\text{Loss of weight in water}} = \frac{\text{Weight of body in air}}{\text{Weight in air} - \text{weight in water}} = \frac{W_1}{W_1 - W_2}$

(6) If the loss of weight of a body in water is '*a*' while in liquid is '*b*'

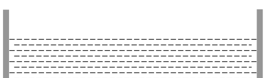
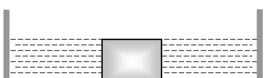
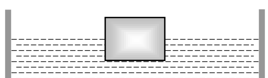
$\therefore \quad \frac{\sigma_L}{\sigma_W} = \frac{\text{Upthrust on body in liquid}}{\text{Upthrust on body in water}} = \frac{\text{Loss of weight in liquid}}{\text{Loss of weight in water}} = \frac{a}{b} = \frac{W_{\text{air}} - W_{\text{liquid}}}{W_{\text{air}} - W_{\text{water}}}$

11.5 Floatation

(1) **Translatory equilibrium :** When a body of density ρ and volume *V* is immersed in a liquid of density σ, the forces acting on the body are

Weight of body $W = mg = V\rho g,$ acting vertically downwards through centre of gravity of the body.

Upthrust force = $V\sigma g$ acting vertically upwards through the centre of gravity of the displaced liquid *i.e.*, centre of buoyancy.

If density of body is greater than that of	If density of body is equal to that of	If density of body is lesser than that of
		

liquid $\rho > \sigma$	liquid $\rho = \sigma$	liquid $\rho < \sigma$
Weight will be more than upthrust so the body will sink	Weight will be equal to upthrust so the body will float fully submerged in neutral equilibrium anywhere in the liquid.	Weight will be less than upthrust so the body will move upwards and in equilibrium will float partially immersed in the liquid Such that, $W = V_{in}\sigma g \Rightarrow$ $V\rho g = V_{in}\sigma g$ $V\rho = V_{in}\sigma$ Where V_{in} is the volume of body in the liquid

Important points

(i) A body will float in liquid only and only if $\rho \le \sigma$

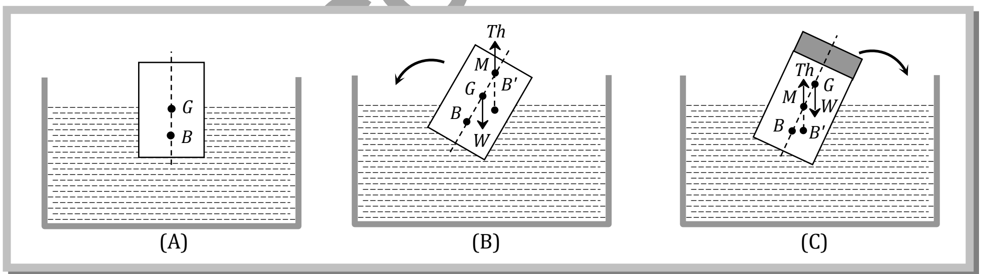
(ii) In case of floating as weight of body = upthrust

So W_{App} = Actual weight – upthrust = 0

(iii) In case of floating $V\rho g = V_{in}\sigma g$

So the equilibrium of floating bodies is unaffected by variations in g though both thrust and weight depend on g.

(2) **Rotatory Equilibrium :** When a floating body is slightly tilted from equilibrium position, the centre of buoyancy B shifts. The vertical line passing through the new centre of buoyancy B' and initial vertical line meet at a point M called meta-centre. If the meta-centre M is above the centre of gravity the couple due to forces at G (weight of body W) and at B' (upthrust) tends to bring the body back to its original position. So for rotational equilibrium of floating body the meta-centre must always be higher than the centre of gravity of the body.



However, if meta-centre goes below CG, the couple due to forces at G and B' tends to topple the floating body.

That is why a wooden log cannot be made to float vertical in water or a boat is likely to capsize if the sitting passengers stand on it. In these situations CG becomes higher than MC and so the body will topple if slightly tilted.

(3) **Application of floatation**

(i) When a body floats then the weight of body = Upthrust

$$V\rho g = V_{in}\sigma g \Rightarrow V_{in} = \left(\frac{\rho}{\sigma}\right)V \quad \therefore \quad V_{out} = V - V_{in} = \left(1 - \frac{\rho}{\sigma}\right)V$$

i.e., Fraction of volume outside the liquid $f_{out} = \dfrac{V_{out}}{V} = \left[1 - \dfrac{\rho}{\sigma}\right]$

(ii) For floatation $V\rho = V_{in}\sigma \Rightarrow \rho = \dfrac{V_{in}}{V}\sigma = f_{in}\sigma$

If two different bodies A and B are floating in the same liquid then $\dfrac{\rho_A}{\rho_B} = \dfrac{(f_{in})_A}{(f_{in})_B}$

(iii) If the same body is made to float in different liquids of densities σ_A and σ_B respectively.

$$V\rho = (V_{in})_A \sigma_A = (V_{in})_B \sigma_B \quad \therefore \quad \frac{\sigma_A}{\sigma_B} = \frac{(V_{in})_B}{(V_{in})_A}$$

(iv) If a platform of mass *M* and cross-section *A* is floating in a liquid of density σ with its height *h* inside the liquid

$$Mg = hA\sigma g \qquad \text{......(i)}$$

Now if a body of mass *m* is placed on it and the platform sinks by *y* then

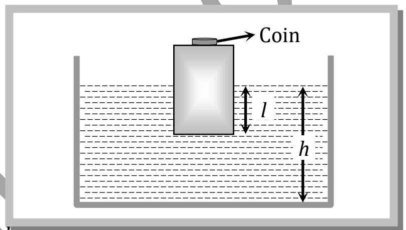
$$(M+m)g = (y+h)A\sigma g \qquad \text{......(ii)}$$

Subtracting equation (i) and (ii), $mg = A\sigma y g$, *i.e.,* $W \propto y$ ......(iii)

So we can determine the weight of a body by placing it on a floating platform and noting the depression of the platform in the liquid by it.

Sample problems based on Archimedes principle

Problem **14.** A wooden block, with a coin placed on its top, floats in water as shown in fig. the distance *l* and *h* are shown there. After some time the coin falls into the water. Then **[IIT-JEE (Screening) 2002]**



(a) *l* decreases and *h* increases

(b) *l* increases and *h* decreases

(c) Both *l* and *h* increase

(d) Both *l* and *h* decrease

Solution : (d) As the block moves up with the fall of coin, *l* decreases, similarly *h* will also decrease because when the coin is in water, it displaces water equal to its own volume only.

Problem **15.** A hemispherical bowl just floats without sinking in a liquid of density $1.2 \times 10^3 kg/m^3$. If outer diameter and the density of the bowl are 1 *m* and $2 \times 10^4 \ kg/m^3$ respectively, then the inner diameter of the bowl will be

(a) 0.94 *m* (b) 0.97 *m* (c) 0.98 *m* (d) 0.99 *m*

Solution : (c) Weight of the bowl = $mg = V\rho g = \frac{4}{3}\pi\left[\left(\frac{D}{2}\right)^3 - \left(\frac{d}{2}\right)^3\right]\rho g$

where *D* is the outer diameter , *d* is the inner diameter and ρ is the density of bowl

Weight of the liquid displaced by the bowl $= V\sigma g = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 \sigma g$

where σ is the density of the liquid.

For the flotation $\frac{4}{3}\pi\left(\frac{D}{2}\right)^3 \sigma g = \frac{4}{3}\pi\left[\left(\frac{D}{2}\right)^3 - \left(\frac{d}{2}\right)^3\right]\rho g \Rightarrow \left(\frac{1}{2}\right)^3 \times 1.2 \times 10^3 = \left[\left(\frac{1}{2}\right)^3 - \left(\frac{d}{2}\right)^3\right] 2 \times 10^4$

By solving we get *d* = 0.98 *m*.

Problem **16.** In making an alloy, a substance of specific gravity s_1 and mass m_1 is mixed with another substance of specific gravity s_2 and mass m_2 ; then the specific gravity of the alloy is **[CPMT 1995]**

(a) $\left(\frac{m_1+m_2}{s_1+s_2}\right)$ (b) $\left(\frac{s_1 s_2}{m_1+m_2}\right)$ (c) $\frac{m_1+m_2}{\left(\frac{m_1}{s_1}+\frac{m_2}{s_2}\right)}$ (d) $\frac{\left(\frac{m_1}{s_1}+\frac{m_2}{s_2}\right)}{m_1+m_2}$

Solution : (c) Specific gravity of alloy $= \frac{\text{Density of alloy}}{\text{Density of water}} = \frac{\text{Mass of alloy}}{\text{Volume of alloy} \times \text{density of water}}$

$$= \frac{m_1 + m_2}{\left(\frac{m_1}{\rho_1} + \frac{m_2}{p_2}\right) \times \rho_w} = \frac{m_1 + m_2}{\frac{m_1}{\rho_1 / \rho_w} + \frac{m_2}{\rho_2 / \rho_w}} = \frac{m_1 + m_2}{\frac{m_1}{s_1} + \frac{m_2}{s_2}}$$

$$\left[\text{As specific gravity of substance} = \frac{\text{density of substance}}{\text{density of water}}\right]$$

Problem 17. A concrete sphere of radius R has a cavity of radius r which is packed with sawdust. The specific gravities of concrete and sawdust are respectively 2.4 and 0.3 for this sphere to float with its entire volume submerged under water. Ratio of mass of concrete to mass of sawdust will be **[AIIMS 1995]**

(a) 8 (b) 4 (c) 3 (d) Zero

Solution : (b) Let specific gravities of concrete and saw dust are ρ_1 and ρ_2 respectively.

According to principle of floatation weight of whole sphere = upthrust on the sphere

$$\frac{4}{3}\pi(R^3 - r^3)\rho_1 g + \frac{4}{3}\pi r^3 \rho_2 g = \frac{4}{3}\pi R^3 \times 1 \times g \Rightarrow R^3\rho_1 - r^3\rho_1 + r^3\rho_2 = R^3$$

$$\Rightarrow \quad R^3(\rho_1 - 1) = r^3(\rho_1 - \rho_2) \quad \Rightarrow \quad \frac{R^3}{r^3} = \frac{\rho_1 - \rho_2}{\rho_1 - 1} \quad \Rightarrow \quad \frac{R^3 - r^3}{r^3} = \frac{\rho_1 - \rho_2 - \rho_1 + 1}{\rho_1 - 1} \Rightarrow$$

$$\frac{(R^3 - r^3)\rho_1}{r^3\rho_2} = \left(\frac{1 - \rho_2}{\rho_1 - 1}\right)\frac{\rho_1}{\rho_2}$$

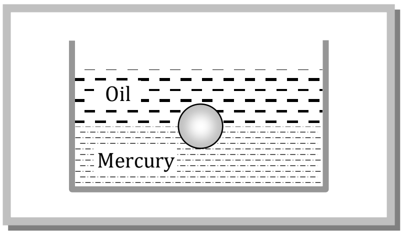
$$\Rightarrow \frac{\text{Mass of concrete}}{\text{Mass of saw dust}} = \left(\frac{1 - 0.3}{2.4 - 1}\right) \times \frac{2.4}{0.3} = 4$$

Problem 18. A vessel contains oil (density = 0.8 gm/cm^3) over mercury (density = 13.6 gm/cm^3). A homogeneous sphere floats with half of its volume immersed in mercury and the other half in oil. The density of the material of the sphere in gm/cm^3 is **[IIT-JEE 1988]**

(a) 3.3 (b) 6.4 (c) 7.2 (d) 12.8

Solution : (c) As the sphere floats in the liquid. Therefore its weight will be equal to the upthrust force on it

Weight of sphere $= \frac{4}{3}\pi R^3 \rho g$ ...... (i)

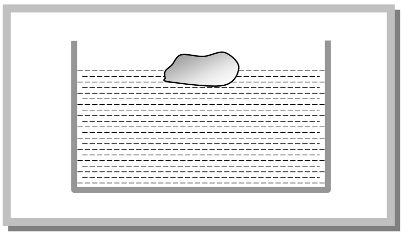


Upthrust due to oil and mercury $= \frac{2}{3}\pi R^3 \times \sigma_{oil} g + \frac{2}{3}\pi R^3 \sigma_{Hg} g$ ......(ii)

Equating (i) and (ii)

$$\frac{4}{3}\pi R^3 \rho g = \frac{2}{3}\pi R^3 0.8 g + \frac{2}{3}\pi R^3 \times 13.6 g \Rightarrow 2\rho = 0.8 + 13.6 = 14.4 \Rightarrow \rho = 7.2$$

Problem 19. A body floats in a liquid contained in a beaker. The whole system as shown falls freely under gravity. The upthrust on the body due to the liquid is **[IIT-JEE1982]**



(a) Zero

(b) Equal to the weight of the liquid displaced

(c) Equal to the weight of the body in air

(d) Equal to the weight of the immersed position of the body

Solution : (a) Upthrust $= V\rho_{\text{liquid}}(g - a)$; where, a = downward acceleration, V = volume of liquid displaced

But for free fall $a = g$ $\therefore$ Upthrust = 0

Problem 20. A metallic block of density 5 gm cm^{-3} and having dimensions 5 cm × 5 cm × 5 cm is weighed in water. Its apparent weight will be

(a) 5 × 5 × 5 × 5 gf (b) 4 × 4 × 4 × 4 gf (c) 5 × 4 × 4 × 4 gf (d) 4 × 5 × 5 × 5 gf

Solution : (d) Apparent weight $= V(\rho - \sigma)g = l \times b \times h \times (5 - 1) \times g = 5 \times 5 \times 5 \times 4 \times g$ *Dyne* or $4 \times 5 \times 5 \times 5$ gf.

Problem 21. A wooden block of volume 1000 cm^3 is suspended from a spring balance. It weighs 12 N in air. It is suspended in water such that half of the block is below the surface of water. The reading of the spring balance is

(a) 10 N (b) 9 N (c) 8 N (d) 7 N

Solution : (d) Reading of the spring balance = Apparent weight of the block = Actual weight – upthrust

$$= 12 - V_{in}\sigma g = 12 - 500 \times 10^{-6} \times 10^{3} \times 10 = 12 - 5 = 7N.$$

***Problem* 22.** An iceberg is floating in sea water. The density of ice is 0.92 gm/cm^3 and that of sea water is 1.03g/cm^3. What percentage of the iceberg will be below the surface of water

(a) 3% (b) 11% (c) 89% (d) 92%

Solution : (c) For the floatation of ice-berg, Weight of ice = upthrust due to displaced water

$$V\rho g = V_{in}\sigma g \Rightarrow V_{in} = \left(\frac{\rho}{\sigma}\right)V = \left(\frac{0.92}{1.03}\right)V = 0.89\,V \quad \therefore \frac{V_{in}}{V} = 0.89 \text{ or } 89\%.$$

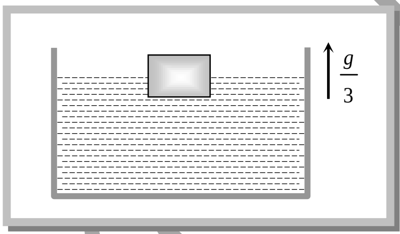
***Problem* 23.** A cubical block is floating in a liquid with half of its volume immersed in the liquid. When the whole system accelerates upwards with acceleration of *g/3*, the fraction of volume immersed in the liquid will be

(a) $\frac{1}{2}$

(b) $\frac{3}{8}$

(c) $\frac{2}{3}$

(d) $\frac{3}{4}$



Solution : (a) Fraction of volume immersed in the liquid $V_{in} = \left(\frac{\rho}{\sigma}\right)V$ *i.e.* it depends upon the densities of the block and liquid.

So there will be no change in it if system moves upward or downward with constant velocity or some acceleration.

***Problem* 24.** A silver ingot weighing 2.1 *kg* is held by a string so as to be completely immersed in a liquid of relative density 0.8. The relative density of silver is 10.5. The tension in the string in *kg-wt* is

(a) 1.6 (b) 1.94 (c) 3.1 (d) 5.25

Solution : (b) Apparent weight $= V(\rho - \sigma)g = \frac{M}{\rho}(\rho - \sigma)g = M\left(1 - \frac{\sigma}{\rho}\right)g = 2.1\left(1 - \frac{0.8}{10.5}\right)g = 1.94\,g$ *Newton* = 1.94 *Kg-wt*

***Problem* 25.** A sample of metal weighs 210 *gm* in air, 180 *gm* in water and 120 *gm* in liquid. Then relative density (RD) of

(a) Metal is 3 (b) Metal is 7 (c) Liquid is 3 (d) Liquid is $\frac{1}{3}$

Solution : (b, c) Let the density of metal is ρ and density of liquid is σ.

If *V* is the volume of sample then according to problem

$210 = V\rho g$ ....(i)

$180 = V(\rho - 1)g$ ....(ii)

$120 = V(\rho - \sigma)g$ ....(iii)

By solving (i), (ii) and (iii) we get $\rho = 7$ and $\sigma = 3$.

***Problem* 26.** Two solids *A* and *B* float in water. It is observed that *A* floats with half its volume immersed and *B* floats with 2/3 of its volume immersed. Compare the densities of *A* and *B*

(a) 4 : 3 (b) 2 : 3 (c) 3 : 4 (d) 1 : 3

Solution : (c) If two different bodies *A* and *B* are floating in the same liquid then $\frac{\rho_A}{\rho_B} = \frac{(f_{in})_A}{(f_{in})_B} = \frac{1/2}{2/3} = \frac{3}{4}$

***Problem* 27.** The fraction of a floating object of volume V_0 and density d_0 above the surface of a liquid of density *d* will be

(a) $\frac{d_0}{d}$ (b) $\frac{dd_0}{d + d_0}$ (c) $\frac{d - d_0}{d}$ (d) $\frac{dd_0}{d - d_0}$

Solution : (c) For the floatation $V_0 d_0 g = V_{in}\,d\,g \Rightarrow V_{in} = V_0\frac{d_0}{d}$

$$\therefore \quad V_{out} = V_0 - V_{in} = V_0 - V_0 \frac{d_0}{d} = V_0\left[\frac{d - d_0}{d}\right] \Rightarrow \frac{V_{out}}{V_0} = \frac{d - d_0}{d}.$$

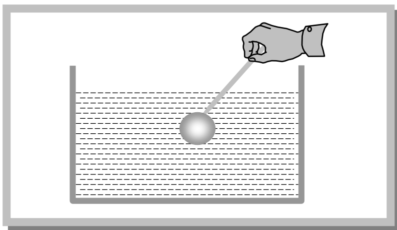
Problem **28.** A vessel with water is placed on a weighing pan and reads 600 *g*. Now a ball of 40 *g* and density 0.80 *g/cc* is sunk into the water with a pin as shown in fig. keeping it sunk. The weighing pan will show a reading

(a) 600 *g*

(b) 550 *g*

(c) 650 *g*

(d) 632 *g*



Solution : (c) Upthrust on ball = weight of displaced water

$$= V\sigma g = \left(\frac{m}{\rho}\right)\sigma g = \frac{40}{0.8} \times 1 \times g = 50\,g \ \textit{Dyne} \text{ or } 50\,gm$$

As the ball is sunk into the water with a pin by applying downward force equal to upthrust on it.

So reading of weighing pan = weight of water + downward force against upthrust = 600 + 50 = 650 *gm*.

11.6 Some Conceptual Questions

***Que*.1** Why a small iron needle sinks in water while a large iron ship floats

Ans. *For floatation, the density of body must be lesser or equal to that of liquid. In case of iron needle, the density of needle, i.e., iron is more than that of water, so it will sink. However, the density of a ship due to its large volume is lesser than that of water, so it will float.*

***Que*.2** A man is sitting in a boat which is floating in a pond. If the man drinks some water from the pond, what will happen to the level of water in the pond

Ans. *If the man drinks m g of water from the pond, the weight of (boat + man) system will increase by mg and so the system will displace mg more water for floating. So due to removal of water from pond, the water level in pond will fall but due to water displaced by the floating system the water level in the pond will rise and so the water removed from the pond is equal to the water displaced by the system; the level of water in the pond will remain unchanged.*

***Que*.3** A boy is carrying a fish in one hand and a bucket full of water in the other hand. He then places the fish in the bucket thinking that in accordance with Archimedes' principle he is now carrying less weight as the weight of the fish will reduce due to upthrust. Is he right

Ans. *No, when he places the fish in water in the bucket, no doubt the weight of fish is reduced due to upthrust, but the weight of (water + bucket) system is increased by the same amount, so that the total weight carried by him remains unchanged.*

***Que*. 4** A bucket of water is suspended from a spring balance. Does the reading of balance change (a) when a piece of stone suspended from a string is immersed in the water without touching the bucket? (b) when a piece of iron or cork is put in the water in the bucket?

Ans. (a) *Yes, the reading of the balance will increase but the increase in weight will be equal to the loss in weight of the stone* $(V\sigma g)$ *and not the weight of stone* $(V\rho g)[> V\sigma g$ as $\rho > \sigma]$.

(b) *Yes, the reading of the balance will increase but the increase in weight will be equal to the weight of iron or cork piece.*

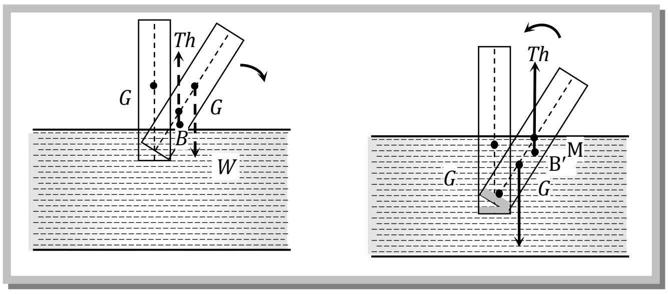
***Que*. 5** Why a soft plastic bag weighs the same when empty or when filled with air at atmospheric pressure? Would the weight be the same if measured in vacuum

Ans. *If the weight of empty bag is* W_0 *and the volume of bag is V, when the bag is filled with air of density* ρ *at NTP, its weights will increase by* $V\rho g$. *Now when the bag filled with air is weighed in air, the thrust of air* $V\rho g$ *will decrease its weight; so* $W = W_0 + V\rho g - V\rho g = W_0$

i.e., the weight of the bag remains unchanged when it is filled with air at NTP and weighed in air. However if the bag is weighed in vacuum will be W_0 *when empty and* $(W_0 + V\rho g)$ *when filled with air (as there is no upthrust), i.e., in vacuum an air-filled bag will weigh more than an empty bag.*

***Que*.6** Why does a uniform wooden stick or log float horizontally? If enough iron is added to one end, it will float vertically; explain this also.

Ans. *When a wooden stick is made to float vertically, its rotational equilibrium will be unstable as its meta-centre will be lower than its CG and with a slight tilt it will rotate under the action of the couple formed by thrust and weight in the direction of tilt, till it becomes horizontal.*



However, due to loading at the bottom, the CG of the stick (or log) will be lowered and so may be lower than the meta-centre. In this situation the equilibrium will be stable and if the stick (or log) is tilted, it will come back to its initial vertical position.

Que.7 A boat containing some pieces of material is floating in a pond. What will happen to the level of water in the pond if on unloading the pieces in the pond, the piece (*a*) floats (*b*) sinks?

Ans. *If M is the mass of boat and m of pieces in it, then initially as the system is floating* $M + m = V_D \sigma_W$

i.e., the system displaces water $V_D = \frac{M}{\sigma_W} + \frac{m}{\sigma_W}$ ......(i)

When the pieces are dropped in the pond, the boat will still float, so it displaces water $M = V_1 \sigma_W$,

i.e, $V_1 = (M / \sigma_W)$

(a) Now if the unloaded pieces floats in the pond, the water displaced by them $m = V_2 \sigma_W$,

i.e, $V_2 = (m / \sigma_W)$

So the total water displaced by the boat and the floating pieces

$$V_1 + V_2 = \frac{M}{\sigma_W} + \frac{m}{\sigma_W} \quad ......(ii)$$

Which is same as the water displaced by the floating system initially (eqn. 1); so the level of water in the pond will remain unchanged.

(b) Now if the unloaded pieces sink the water displaced by them will be equal to their own volume, i.e,

$$V_2' = \frac{m}{\rho} \quad \left[as \;\; \rho = \frac{m}{V}\right]$$

and so in this situation the total volume of water displaced by boat and sinking pieces will be

$$V_1 + V_2' = \left(\frac{M}{\sigma_W} + \frac{m}{\rho}\right) \quad .....(iii)$$

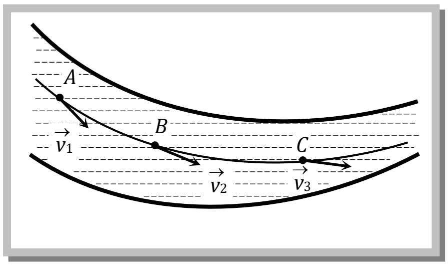
Now as the pieces are sinking $\rho > \sigma_W$, *so this volume will be lesser than initial water displaced by the floating system (eq. 1); so the level of water in the pond will go down (or fall)*

In this problem if the pieces (either sinking or floating) are unloaded on the ground, the water displaced after unloading, $V_2 = M / \sigma_W$, *will be lesser than before unloading.* $V = (M + m) / \sigma_W$; *so the level of water in the pond will fall.*

11.7 Streamline, Laminar and Turbulent Flow

(1) **Stream line flow :** Stream line flow of a liquid is that flow in which each element of the liquid passing through a point travels along the same path and with the same velocity as the preceeding element passes through that point.

A streamline may be defined as the path, straight or curved, the tangent to which at any point gives the direction of the flow of liquid at that point.



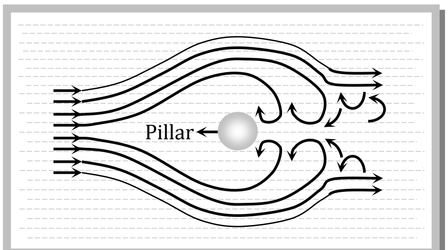
The two streamlines cannot cross each other and the greater is the crowding of streamlines at a place, the greater is the velocity of liquid particles at that place.

Path *ABC* is streamline as shown in the figure and v_1, v_2 and v_3 are the velocities of the liquid particles at *A*, *B* and *C* point respectively.

(2) **Laminar flow :** If a liquid is flowing over a horizontal surface with a steady flow and moves in the form of layers of different velocities which do not mix with each other, then the flow of liquid is called laminar flow.

In this flow the velocity of liquid flow is always less than the critical velocity of the liquid. The laminar flow is generally used synonymously with streamlined flow.

(3) **Turbulent flow :** When a liquid moves with a velocity greater than its critical velocity, the motion of the particles of liquid becomes disordered or irregular. Such a flow is called a turbulent flow.



In a turbulent flow, the path and the velocity of the particles of the liquid change continuously and haphazardly with time from point to point. In a turbulent flow, most of the external energy maintaining the flow is spent in producing eddies in the liquid and only a small fraction of energy is available for forward flow. For example, eddies are seen by the sides of the pillars of a river bridge.

11.8 Critical Velocity and Reynold's Number

The critical velocity is that velocity of liquid flow upto which its flow is streamlined and above which its flow becomes turbulent.

Reynold's number is a pure number which determines the nature of flow of liquid through a pipe.

It is defined as the ratio of the inertial force per unit area to the viscous force per unit area for a flowing fluid.

$$N_R = \frac{\text{Inertial force per unit area}}{\text{Viscous force per unit area}}$$

If a liquid of density ρ is flowing through a tube of radius *r* and cross section *A* then mass of liquid flowing through the tube per second $\frac{dm}{dt}$ = volume flowing per second × density = $Av \times \rho$

$\therefore$ Inertial force per unit area = $\frac{dp/dt}{A} = \frac{v(dm/dt)}{A} = \frac{v\,Av\,\rho}{A} = v^2\rho$

Viscous force per unit area $F/A = \frac{\eta v}{r}$

So by the definition of Reynolds number $N_R = \frac{\text{Inertial force per unit area}}{\text{Viscous force per unit area}} = \frac{v^2\rho}{\eta v/r} = \frac{v\rho r}{\eta}$

If the value of Reynold's number

(i) Lies between 0 to 2000, the flow of liquid is streamline or laminar.

(ii) Lies between 2000 to 3000, the flow of liquid is unstable changing from streamline to turbulent flow.

(iii) Above 3000, the flow of liquid is definitely turbulent.

Sample problems based on Stream lined and Turbulent flow

***Problem* 29.** In which one of the following cases will the liquid flow in a pipe be most streamlined

(a) Liquid of high viscosity and high density flowing through a pipe of small radius

(b) Liquid of high viscosity and low density flowing through a pipe of small radius

(c) Liquid of low viscosity and low density flowing through a pipe of large radius

(d) Liquid of low viscosity and high density flowing through a pipe of large radius

Solution : (b) For streamline flow Reynold's number $N_R \propto \frac{r\rho}{\eta}$ should be less.

For less value of N_R, radius and density should be small and viscosity should be high.

***Problem* 30.** Two different liquids are flowing in two tubes of equal radius. The ratio of coefficients of viscosity of liquids is 52:49 and the ratio of their densities is 13:1, then the ratio of their critical velocities will be

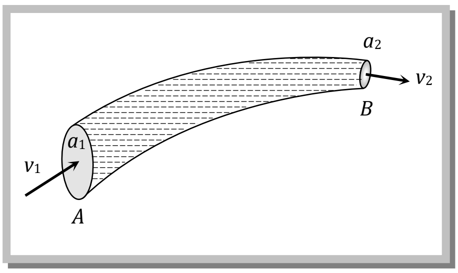
(a) 4 : 49 (b) 49 : 4 (c) 2 : 7 (d) 7 : 2

Solution : (a) Critical velocity $v = N_R \frac{\eta}{\rho r} \Rightarrow \frac{v_1}{v_2} = \frac{\eta_1}{\eta_2} \times \frac{\rho_2}{\rho_1} = \frac{52}{49} \times \frac{1}{13} = \frac{4}{49}$.

11.9 Equation of Continuity

The equation of continuity is derived from the principle of conservation of mass.

A non-viscous liquid in streamline flow passes through a tube *AB* of varying cross section. Let the cross sectional area of the pipe at points *A* and *B* be a_1 and a_2 respectively. Let the liquid enter with normal velocity v_1 at *A* and leave with velocity v_2 at *B*. Let ρ_1 and ρ_2 be the densities of the liquid at point *A* and *B* respectively.



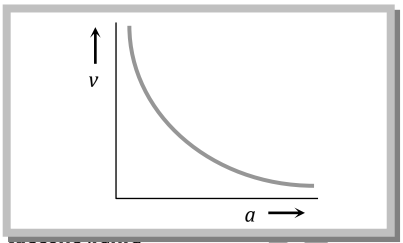
Mass of the liquid entering per second at *A* = Mass of the liquid leaving per second at *B*

$a_1 v_1 \rho_1 = a_2 v_2 \rho_2$

$a_1 v_1 = a_2 v_2$ [If the liquid is incompressible $(\rho_1 = \rho_2)$]

or $av = \text{constant}$

or $a \propto \frac{1}{v}$

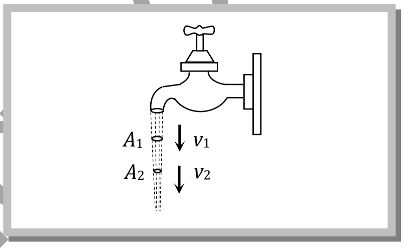


This expression is called the equation of continuity for the steady flow of an incompressible and non-viscous liquid.

(1) The velocity of flow is independent of the liquid (assuming the liquid to be non-viscous)

(2) The velocity of flow will increase if cross-section decreases and vice-versa. That is why :

(a) In hilly region, where the river is narrow and shallow (*i.e.,* small cross-section) the water current will be faster, while in plains where the river is wide and deep (*i.e.*, large cross-section) the current will be slower, and so deep water will appear to be still.



(b) When water falls from a tap, the velocity of falling water under the action of gravity will increase with distance from the tap (*i.e,* $v_2 > v_1$). So in accordance with continuity equation the cross section of the water stream will decrease (*i.e.,* $A_2 < A_1$), *i.e.,* the falling stream of water becomes narrower.

Sample problems based on Equation of continuity

<u>*Problem*</u> 31. Two water pipes of diameters 2 *cm* and 4 *cm* are connected with the main supply line. The velocity of flow of water in the pipe of 2 *cm* diameter is **[MNR 1980]**

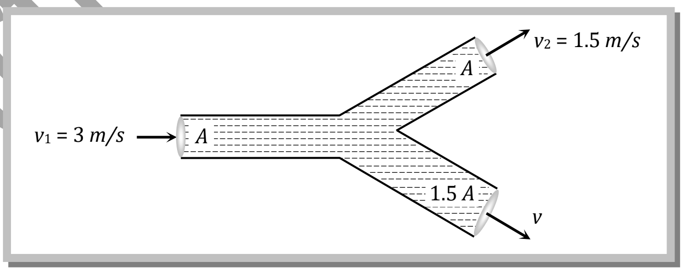
(a) 4 times that in the other pipe (b) $\frac{1}{4}$ times that in the other pipe

(c) 2 times that in the other pipe (d) $\frac{1}{2}$ times that in the other pipe

Solution : (a) $d_A = 2\,cm$ and $d_B = 4\ cm$ $\therefore$ $r_A = 1\ cm$ and $r_B = 2\ cm$

From equation of continuity $av = \text{constant}$ $\therefore \frac{v_A}{v_B} = \frac{a_B}{a_A} = \frac{\pi(r_B)^2}{\pi(r_A)^2} = \left(\frac{2}{1}\right)^2 \Rightarrow v_A = 4v_B$

<u>*Problem*</u> 32. An incompressible liquid flows through a horizontal tube as shown in the following fig. Then the velocity *v* of the fluid is



(a) 3.0 *m/s* (b) 1.5 *m/s* (c) 1.0 *m/s* (d) 2.25 *m/s*

Solution : (c) If the liquid is incompressible then mass of liquid entering through left end, should be equal to mass of liquid coming out from the right end.

$\therefore M = m_1 + m_2 \Rightarrow Av_1 = Av_2 + 1.5A.v \Rightarrow A \times 3 = A \times 1.5 + 1.5A.v \Rightarrow v = 1\ m/s$

<u>*Problem*</u> 33. Water enters through end *A* with speed v_1 and leaves through end B with speed v_2 of a cylindrical tube AB. The tube is always completely filled with water. In case I tube is horizontal and in case II it is vertical with end A upwards and in case III it is vertical with end *B* upwards. We have $v_1 = v_2$ for

(a) Case I (b) Case II (c) Case III (d) Each case

Solution : (d) This happens in accordance with equation of continuity and this equation was derived on the principle of conservation of mass and it is true in every case, either tube remain horizontal or vertical.

<u>*Problem*</u> 34. Water is moving with a speed of 5.18 ms^{-1} through a pipe with a cross-sectional area of 4.20 cm^2. The water gradually descends 9.66 *m* as the pipe increase in area to 7.60 cm^2. The speed of flow at the lower level is

(a) 3.0 ms^{-1} (b) 5.7 ms^{-1} (c) 3.82 ms^{-1} (d) 2.86 ms^{-1}

Solution : (d) $a_1 v_1 = a_2 v_2 \Rightarrow 4.20 \times 5.18 = 7.60 \times v_2 \Rightarrow v_2 = 2.86\ m/s$

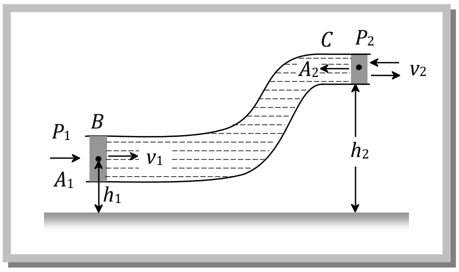
11.10 Energy of a Flowing Fluid

A flowing fluid in motion possesses the following three types of energy

Pressure Energy	Potential energy	Kinetic energy
It is the energy possessed by a liquid by virtue of its pressure. It is the measure of work done in pushing the liquid against pressure without imparting any velocity to it.	It is the energy possessed by liquid by virtue of its height or position above the surface of earth or any reference level taken as zero level.	It is the energy possessed by a liquid by virtue of its motion or velocity.
Pressure energy of the liquid PV	Potential energy of the liquid mgh	Kinetic energy of the liquid $\frac{1}{2}mv^2$
Pressure energy per unit mass of the liquid $\frac{P}{\rho}$	Potential energy per unit mass of the liquid gh	Kinetic energy per unit mass of the liquid $\frac{1}{2}v^2$
Pressure energy per unit volume of the liquid P	Potential energy per unit volume of the liquid ρgh	Kinetic energy per unit volume of the liquid $\frac{1}{2}\rho v^2$

11.11 Bernoulli's Theorem

According to this theorem the total energy (pressure energy, potential energy and kinetic energy) per unit volume or mass of an incompressible and non-viscous fluid in steady flow through a pipe remains constant throughout the flow, provided there is no source or sink of the fluid along the length of the pipe.



Mathematically for unit volume of liquid flowing through a pipe.

$$P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$$

To prove it consider a liquid flowing steadily through a tube of non-uniform area of cross-section as shown in fig. If P_1 and P_2 are the pressures at the two ends of the tube respectively, work done in pushing the volume V of incompressible fluid from point B to C through the tube will be $W = P_1V - P_2V = (P_1 - P_2)V$ ......(i)

This work is used by the fluid in two ways.

(i) In changing the potential energy of mass m (in the volume V) from mgh_1 to mgh_2,

i.e., $\Delta U = mg(h_2 - h_1)$ ......(ii)

(ii) In changing the kinetic energy from $\frac{1}{2}mv_1^2$ to $\frac{1}{2}mv_2^2$, *i.e.,* $\Delta K = \frac{1}{2}m(v_2^2 - v_1^2)$ ......(iii)

Now as the fluid is non-viscous, by conservation of mechanical energy

$$W = \Delta U + \Delta K$$

i.e., $$(P_1 - P_2)V = mg(h_2 - h_1) + \frac{1}{2}m(v_2^2 - v_1^2)$$

or $$P_1 - P_2 = \rho g(h_2 - h_1) + \frac{1}{2}\rho(v_2^2 - v_1^2)$$ [As $\rho = m/V$]

or $$P_1 + \rho gh_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho gh_2 + \frac{1}{2}\rho v_2^2$$

or $$P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$$

This equation is the so called Bernoulli's equation and represents conservation of mechanical energy in case of moving fluids.

(i) Bernoulli's theorem for unit mass of liquid flowing through a pipe can also be written as:

$$\frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{constant}$$

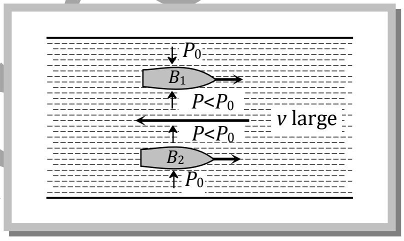
(ii) Dividing above equation by g we get $\frac{P}{\rho g} + h + \frac{v^2}{2g}$ = constant

Here $\frac{P}{\rho g}$ is called pressure head, h is called gravitational head and $\frac{v^2}{2g}$ is called velocity head. From this equation Bernoulli's theorem can be stated as.

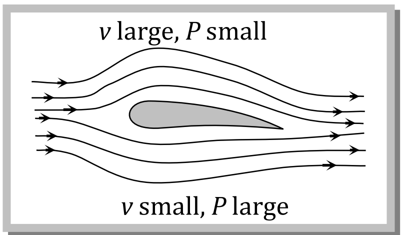
In stream line flow of an ideal liquid, the sum of pressure head, gravitational head and velocity head of every cross section of the liquid is constant.

11.12 Applications of Bernoulli's Theorem

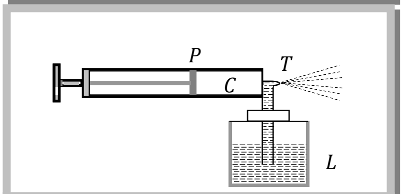
(i) **Attraction between two closely parallel moving boats (or buses) :** When two boats or buses move side by side in the same direction, the water (or air) in the region between them moves faster than that on the remote sides. Consequently in accordance with *Bernoulli's principle* the pressure between them is reduced and hence due to pressure difference they are pulled towards each other creating the so called attraction.



(ii) **Working of an aeroplane :** This is also based on Bernoulli's principle. The wings of the aeroplane are of the shape as shown in fig. Due to this specific shape of wings when the aeroplane runs, air passes at higher speed over it as compared to its lower surface. This difference of air speeds above and below the wings, in accordance with Bernoulli's principle, creates a pressure difference, due to which an upward force called 'dynamic lift' (= pressure difference × area of wing) acts on the plane. If this force becomes greater than the weight of the plane, the plane will rise up.

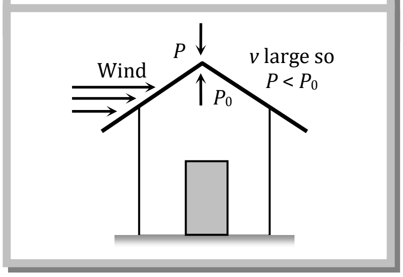


(iii) **Action of atomiser:** The action of carburetor, paint-gun, scent-spray or insect-sprayer is based on Bernoulli's principle. In all these, by means of motion of a piston P in a cylinder C, high speed air is passed over a tube T dipped in liquid L to be sprayed. High speed air creates low pressure over the tube due to which liquid (paint, scent, insecticide or petrol) rises in it and is then blown off in very small droplets with expelled air.



(iv) **Blowing off roofs by wind storms :** During a tornado or hurricane, when a high speed wind blows over a straw or tin roof, it creates a low pressure (P) in accordance with Bernoulli's principle.

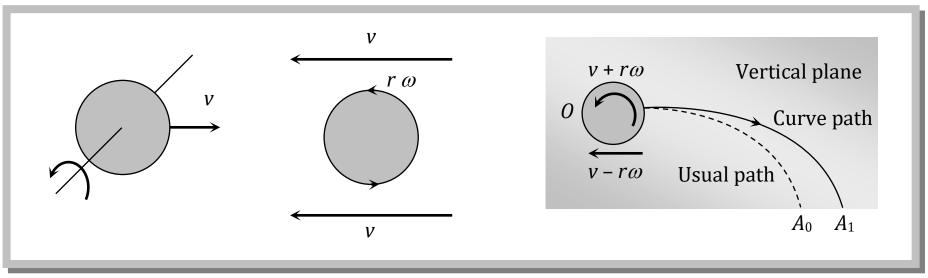
However, the pressure below the roof (*i.e.,* inside the room) is still atmospheric ($= P_0$). So due to this difference of pressure the roof is lifted up and is then blown off by the wind.



(v) **Magnus effect :** When a spinning ball is thrown, it deviates from its usual path in flight. This effect is called Magnus effect and plays as important role in tennis, cricket and soccer, etc. as by applying appropriate spin the moving ball can be made to curve in any desired direction.

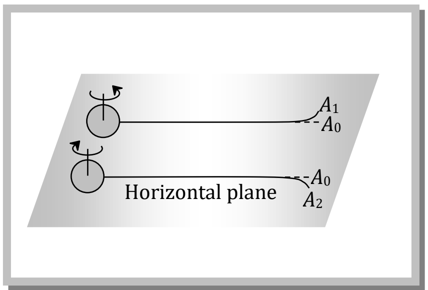
If a ball is moving from left to right and also spinning about a horizontal axis perpendicular to the direction of motion as shown in fig. then relative to the ball, air will be moving from right to left.

The resultant velocity of air above the ball will be $(v + r\omega)$ while below it $(v - r\omega)$. So in accordance with Bernoulli's principle pressure above the ball will be less than below it. Due to this difference of pressure an upward force will act on the ball and hence the ball will deviate from its usual path OA_0 and will hit the ground at A_1 following the path OA_1 *i.e.,* if a ball is thrown with back-spin, the pitch will curve less sharply prolonging the flight.



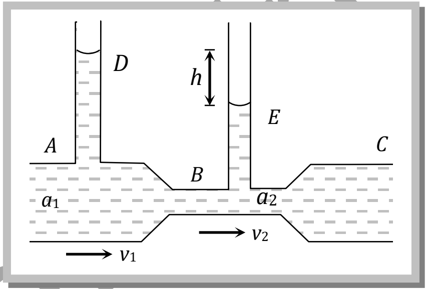
Similarly if the spin is clockwise *i.e.,* the ball is thrown with top-spin, the force due to pressure difference will act in the direction of gravity and so the pitch will curve more sharply shortening the flight.

Furthermore, if the ball is spinning about a vertical axis, the curving will be sideways as shown in producing the so called out swing or in swing.



(vi) **Venturimeter :** It is a device based on Bernoulli's theorem used for measuring the rate of flow of liquid through pipes.

It consists of two identical coaxial tubes *A* and *C* connected by a narrow co-axial tube *B*. Two vertical tubes *D* and *E* are mounted on the tubes *A* and *B* to measure the pressure of the following liquid.



When the liquid flows in the tube *ABC*, the velocity of flow in part *B* will be larger than in the tube *A* or *C*. So the pressure in part *B* will be less than that in tube *A* or *C*. By measuring the pressure difference between *A* and *B*, the rate of flow of the liquid in the tube can be calculated.

Let a_1, a_2 = area of cross section of tube A and B respectively

v_1, v_2 = Velocity of flow of liquid through A and B respectively

P_1, P_2 = Liquid pressure at A and B respectively

$\therefore \quad P_1 - P_2 = h\rho g$ ......(i) [ρ = density of flowing liquid]

From Bernoulli's theorem for horizontal flow of liquid

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) \quad ......(ii)$$

From (i) and (ii) $h\rho g = \frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2}\rho\left[\frac{V^2}{a_2^2} - \frac{V^2}{a_1^2}\right]$ [As $V = a_1 v_1 = a_2 v_2$]

$$\therefore \quad V^2 = \frac{2a_1^2 a_2^2 hg}{a_1^2 - a_2^2} \text{ or } V = a_1 a_2 \sqrt{\frac{2hg}{a_1^2 - a_2^2}}$$

Sample problems based on Bernoulli's Theorem

***Problem* 35.** The velocity of kerosene oil in a horizontal pipe is 5 m/s. If $g = 10 m/s^2$ then the velocity head of oil will be

(a) 1.25 *m* (b) 12.5 *m* (c) 0.125 *m* (d) 125 *m*

Solution : (a) Velocity head $h = \frac{v^2}{2g} = \frac{(5)^2}{2 \times 10} = 1.25\ m$

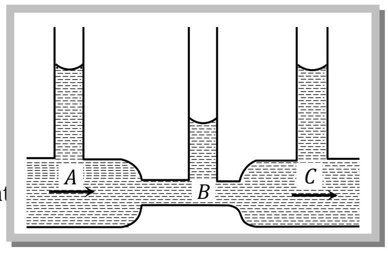
***Problem* 36.** In the following fig. is shown the flow of liquid through a horizontal pipe. Three tubes *A*, *B* and *C* are connected to the pipe. The radii of the tubes *A*, *B* and *C* at the junction are respectively 2 *cm*, 1 *cm* and 2 *cm*. It can be said that the

(a) Height of the liquid in the tube *A* is maximum

(b) Height of the liquid in the tubes *A* and *B* is the same

(c) Height of the liquid in all the three tubes is the same

(d) Height of the liquid in the tubes *A* and *C* is the same

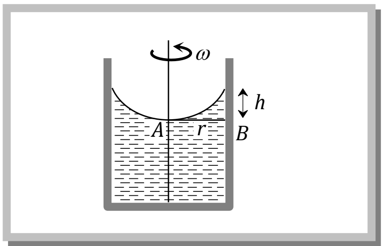


Solution : (d) As cross-section areas of both the tubes *A* and *C* are same and tube is horizont continuity $v_A = v_C$ and therefore according to Bernoulli's theorem $P_A = P_C$ *i.e.* tubes *A* and *C*.

Problem 37. A liquid is kept in a cylindrical vessel which is being rotated about a vertical axis through the centre of the circular base. If the radius of the vessel is r and angular velocity of rotation is ω, then the difference in the heights of the liquid at the centre of the vessel and the edge is

(a) $\frac{r\omega}{2g}$ (b) $\frac{r^2\omega^2}{2g}$ (c) $\sqrt{2gr\omega}$ (d) $\frac{\omega^2}{2gr^2}$

Solution : (b) From Bernoulli's theorem, $P_A + \frac{1}{2}dv_A^2 + dgh_A = P_B + \frac{1}{2}dv_B^2 + dgh_B$



Here, $h_A = h_B \quad \therefore P_A + \frac{1}{2}dv_A^2 = P_B + \frac{1}{2}dv_B^2 \Rightarrow P_A - P_B = \frac{1}{2}d[v_B^2 - v_A^2]$

Now, $v_A = 0, v_B = r\omega$ and $P_A - P_B = hdg$

$\therefore \; hdg = \frac{1}{2}dr^2\omega^2$ or $h = \frac{r^2\omega^2}{2g}$

Problem 38. A manometer connected to a closed tap reads $3.5 \times 10^5\ N/m^2$. When the valve is opened, the reading of manometer falls to $3.0 \times 10^5\ N/m^2$, then velocity of flow of water is

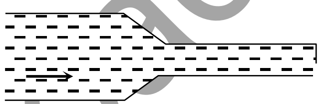
(a) 100 *m/s* (b) 10 *m/s* (c) 1 *m/s* (d) $10\sqrt{10}$ *m/s*

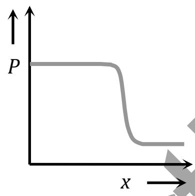
Solution : (b) Bernoulli's theorem for unit mass of liquid $\frac{P}{\rho} + \frac{1}{2}v^2 =$ constant

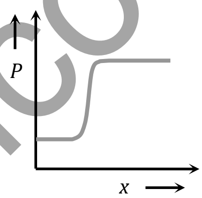
As the liquid starts flowing, it pressure energy decreases $\frac{1}{2}v^2 = \frac{P_1 - P_2}{\rho}$

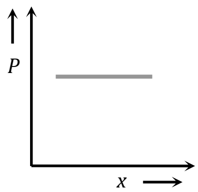
$\Rightarrow \frac{1}{2}v^2 = \frac{3.5 \times 10^5 - 3 \times 10^5}{10^3} \Rightarrow v^2 = \frac{2 \times 0.5 \times 10^5}{10^3} \Rightarrow v^2 = 100 \Rightarrow v = 10\ m/s$

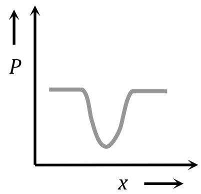
Problem 39. Water flows through a frictionless duct with a cross-section varying as shown in fig. Pressure p at points along the axis is represented by



(a)


(b)


(c)


(d)


Solution : (a) When cross section of duct decreases the velocity of water increases and in accordance with Bernoulli's theorem the pressure decreases at that place.

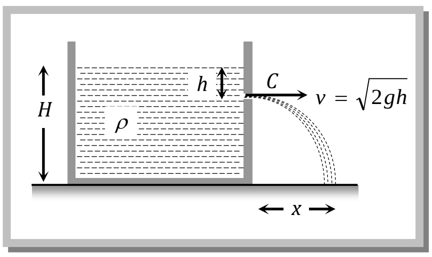
Problem 40. Air is streaming past a horizontal air plane wing such that its speed in 120 *m/s* over the upper surface and 90 *m/s* at the lower surface. If the density of air is 1.3 *kg* per metre^3 and the wing is 10 *m* long and has an average width of 2 *m*, then the difference of the pressure on the two sides of the wing of

(a) 4095.0 *Pascal* (b) 409.50 *Pascal* (c) 40.950 *Pascal* (d) 4.0950 *Pascal*

Solution : (a) From the Bernoulli's theorem $P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2} \times 1.3 \times [(120)^2 - (90)^2] = 4095\ N/m^2$ or *Pascal*

11.13 Velocity of Efflux

If a liquid is filled in a vessel up to height H and a hole is made at a depth h below the free surface of the liquid as shown in fig. then taking the level of hole as reference level (*i.e.,* zero point of potential energy) and applying Bernoulli's principle to the liquid just inside and outside the hole (assuming the liquid to be at rest inside) we get



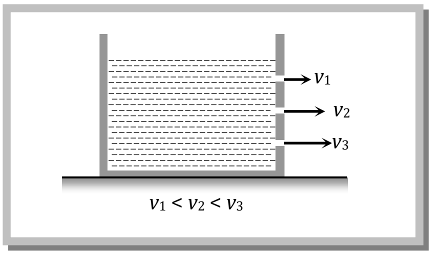
$$\therefore \quad (P_0 + h\rho g) + 0 = P_0 + \frac{1}{2}\rho v^2 \text{ or } v = \sqrt{2gh}$$

Which is same as the speed that an object would acquire in falling from rest through a distance h and is called velocity of efflux or velocity of flow.

This result was first given by Torricelli so this is known as Torricelli's theorem.

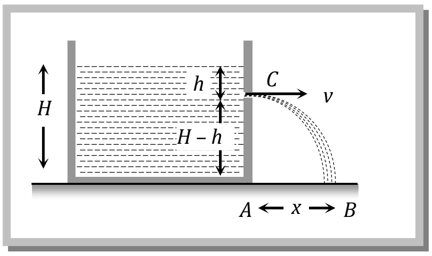
(i) The velocity of efflux is independent of the nature of liquid, quantity of liquid in the vessel and the area of orifice.

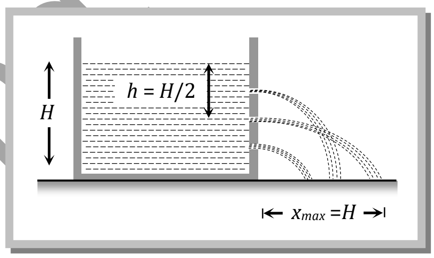
(ii) Greater is the distance of the hole from the free surface of liquid greater will be the velocity of efflux [*i.e.,* $v \propto \sqrt{h}$]



(iii) As the vertical velocity of liquid at the orifice is zero and it is at a height $(H-h)$ from the base, the time taken by the liquid to reach the base-level $t=\sqrt{\frac{2(H-h)}{g}}$

(iv) Now during time *t* liquid is moving horizontally with constant velocity *v*, so it will hit the base level at a horizontal distance *x* (called range) as shown in fig.



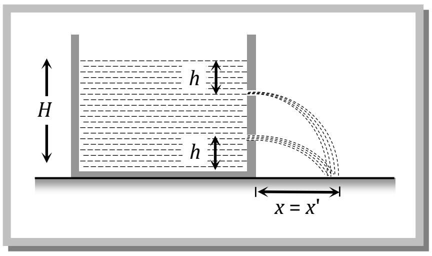


Such that $x = vt = \sqrt{2gh} \times \sqrt{[2(H-h)/g]} = 2\sqrt{h(H-h)}$

For maximum range $\frac{dx}{dh}=0 \quad \therefore \quad h=\frac{H}{2}$

i.e., range *x* will be maximum when $h=\frac{H}{2}$.

$\therefore$ Maximum range $x_{\max} = 2\sqrt{\frac{H}{2}\left[H-\frac{H}{2}\right]} = H$



(v) If the level of free surface in a container is at height *H* from the base and there are two holes at depth *h* and *y* below the free surface, then

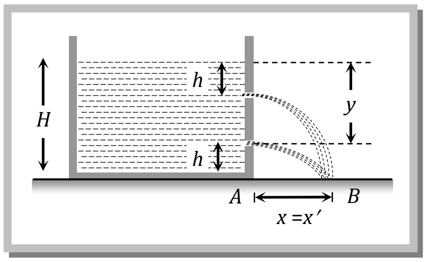
$x = 2\sqrt{h(H-h)}$ and $x' = 2\sqrt{y(H-y)}$

Now if $x = x'$, *i.e.,* $h(H-h) = y(H-y)$

i.e., $y^2 - Hy + h(H-h) = 0$

or $y = \frac{1}{2}[H \pm (H-2h)]$,

i.e., $y = h$ or $(H-h)$

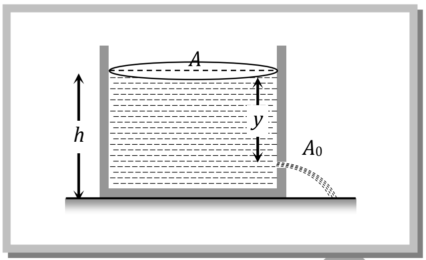


i.e., the range will be same if the orifice is at a depth *h* or $(H-h)$ below the free surface. Now as the distance $(H-h)$ from top means $H-(H-h)=h$ from the bottom, so the range is same for liquid coming out of holes at same distance below the top and above the bottom.

(vi) If A_0 is the area of orifice at a depth *y* below the free surface and *A* that of container, the volume of liquid coming out of the orifice per second will be $(dV/dt) = vA_0 = A_0\sqrt{2gy}$ [As $v = \sqrt{2gy}$]

Due to this, the level of liquid in the container will decrease and so if the level of liquid in the container above the hole changes from y to $y - dy$ in time t to $t + dt$ then $-dV = A\,dy$

So substituting this value of dV in the above equation



$$-A\frac{dy}{dt} = A_0\sqrt{2gy} \;\; i.e., \int dt = -\frac{A}{A_0}\frac{1}{\sqrt{2g}}\int y^{-1/2}dy$$

So the time taken for the level to fall from H to H'

$$t = -\frac{A}{A_0}\frac{1}{\sqrt{2g}}\int_H^{H'} y^{-1/2}dy = \frac{A}{A_0}\sqrt{\frac{2}{g}}[\sqrt{H} - \sqrt{H'}]$$

If the hole is at the bottom of the tank, time t taken to make the tank empty :

$$t = \frac{A}{A_0}\sqrt{\frac{2H}{g}} \qquad \text{[As here } H' = 0\text{]}$$

Sample problems based on Velocity of efflux

***<u>Problem</u>* 41.** A large tank filled with water to a height 'h' is to be emptied through a small hole at the bottom. The ratio of times taken for the level of water to fall from h to $\frac{h}{2}$ and from $\frac{h}{2}$ to zero is **[EAMCET (Engg.) 2003]**

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{2} - 1$ (d) $\frac{1}{\sqrt{2} - 1}$

Solution : (c) Time taken for the level to fall from H to H' $t = \frac{A}{A_0}\sqrt{\frac{2}{g}}\left[\sqrt{H} - \sqrt{H'}\right]$

According to problem- the time taken for the level to fall from h to $\frac{h}{2}$ $t_1 = \frac{A}{A_0}\sqrt{\frac{2}{g}}\left[\sqrt{h} - \sqrt{\frac{h}{2}}\right]$

and similarly time taken for the level to fall from $\frac{h}{2}$ to zero $t_2 = \frac{A}{A_0}\sqrt{\frac{2}{g}}\left[\sqrt{\frac{h}{2}} - 0\right]$

$$\therefore \frac{t_1}{t_2} = \frac{1 - \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}} - 0} = \sqrt{2} - 1.$$

***<u>Problem</u>* 42.** A cylinder of height 20 *m* is completely filled with water. The velocity of efflux of water (in *m/s*) through a small hole on the side wall of the cylinder near its bottom is **[AIEEE 2002]**

(a) 10 (b) 20 (c) 25.5 (d) 5

Solution : (b) $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 20} = 20\ m/s$

***<u>Problem</u>* 43.** There is a hole in the bottom of tank having water. If total pressure at bottom is 3 *atm* (1 *atm* = $10^5 N/m^2$) then the velocity of water flowing from hole is **[CPMT 2002]**

(a) $\sqrt{400}\ m/s$ (b) $\sqrt{600}\ m/s$ (c) $\sqrt{60}\ m/s$ (d) None of these

Solution : (b) Pressure at the bottom of tank $P = h\rho g = 3 \times 10^5 \frac{N}{m^2}$ and velocity of water $v = \sqrt{2gh}$

$$\therefore v = \sqrt{\frac{2P}{\rho}} = \sqrt{\frac{2 \times 3 \times 10^5}{10^3}} = \sqrt{600}\ m/s$$

Problem 44. A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth $4y$ from the top. When the tank is completely filled with water the quantities of water flowing out per second from both the holes are the same. Then R is equal to

[IIT-JEE (Screening) 2000]

(a) $2\pi L$ (b) $\frac{L}{\sqrt{2\pi}}$ (c) L (d) $\frac{L}{2\pi}$

Solution : (b) Velocity of efflux when the hole is at depth h, $v = \sqrt{2gh}$

Rate of flow of water from square hole $Q_1 = a_1 v_1 = L^2 \sqrt{2gy}$

Rate of flow of water from circular hole $Q_2 = a_2 v_2 = \pi R^2 \sqrt{2g(4y)}$

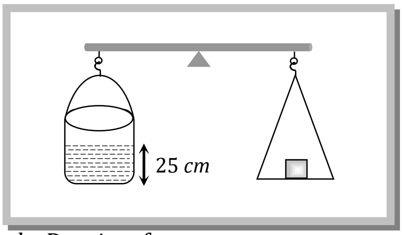
and according to problem $Q_1 = Q_2 \Rightarrow L^2\sqrt{2gy} = \pi R^2 \sqrt{2g(4y)} \Rightarrow R = \frac{L}{\sqrt{2\pi}}$

Problem 45. There is a hole of area A at the bottom of cylindrical vessel. Water is filled up to a height h and water flows out in t second. If water is filled to a height $4h$, it will flow out in time equal to **[MP PMT 1997]**

(a) t (b) $4t$ (c) $2t$ (d) $t/4$

Solution : (c) Time required to emptied the tank $t = \frac{A}{A_0}\sqrt{\frac{2H}{g}} \Rightarrow \frac{t_2}{t_1} = \sqrt{\frac{H_2}{H_1}} = \sqrt{\frac{4h}{h}} = 2 \quad \therefore \; t_2 = 2t$

Problem 46. A cylinder containing water up to a height of 25 cm has a hole of cross-section $\frac{1}{4} cm^2$ in its bottom. It is counterpoised in a balance. What is the initial change in the balancing weight when water begins to flow out



(a) Increase of 12.5 *gm-wt*

(b) Increase of 6.25 *gm-wt*

(c) Decrease of 12.5 *gm-wt*

(d) Decrease of 6.25 *gm-wt*

Solution : (c) Let A = The area of cross section of the hole, v = Initial velocity of efflux, d = Density of water,

Initial volume of water flowing out per second = Av

Initial mass of water flowing out per second = Avd

Rate of change of momentum = Adv^2 $\quad\therefore$ Initial downward force on the out flowing water = Adv^2

So equal amount of reaction acts upwards on the cylinder.

$\therefore$ Initial upward reaction = Adv^2 [As $v = \sqrt{2gh}$]

$\therefore$ Initial decrease in weight $= Ad(2gh) = 2Adgh = 2 \times \left(\frac{1}{4}\right) \times 1 \times 980 \times 25 = 12.5$ *gm-wt.*

Problem 47. A cylindrical tank has a hole of 1 cm^2 in its bottom. If the water is allowed to flow into the tank from a tube above it at the rate of 70 cm^3/sec. then the maximum height up to which water can rise in the tank is

(a) 2.5 cm (b) 5 cm (c) 10 cm (d) 0.25 cm

Solution : (a) The height of water in the tank becomes maximum when the volume of water flowing into the tank per second becomes equal to the volume flowing out per second.

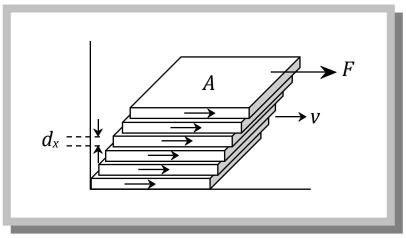
Volume of water flowing out per second = $Av = A\sqrt{2gh}$

and volume of water flowing in per second $= 70\, cm^3/\text{sec}$.

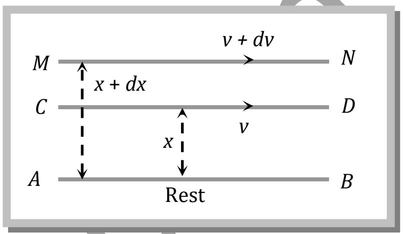
$\therefore \; A\sqrt{2gh} = 70 \Rightarrow 1 \times \sqrt{2gh} = 70 \Rightarrow 1 \times \sqrt{2 \times 980 \times h} = 70 \quad \therefore \; h = \frac{4900}{1960} = 2.5\, cm.$

11.14 Viscosity and Newton's law of Viscous Force

In case of steady flow of a fluid when a layer of fluid slips or tends to slip on adjacent layers in contact, the two layer exert tangential force on each other which tries to destroy the relative motion between them. The property of a fluid due to which it opposes the relative motion between its different layers is called viscosity (or fluid friction or internal friction) and the force between the layers opposing the relative motion is called viscous force.



Consider the two layers CD and MN of the liquid at distances x and $x + dx$ from the fixed surface AB, having the velocities v and $v + dv$ respectively. Then $\frac{dv}{dx}$ denotes the rate of change of velocity with distance and is known as velocity gradient.



According to Newton's hypothesis, the tangential force F acting on a plane parallel layer is proportional to the area of the plane A and the velocity gradient $\frac{dv}{dx}$ in a direction normal to the layer, *i.e.*,

$$F \propto A \quad \text{and} \quad F \propto \frac{dv}{dx} \quad \therefore \quad F \propto A\frac{dv}{dx}$$

or $F = -\eta A \frac{dv}{dx}$

Where η is a constant called the coefficient of viscosity. Negative sign is employed because viscous force acts in a direction opposite to the flow of liquid.

If $A = 1, \frac{dv}{dx} = 1$ then $\eta = F$.

Hence the coefficient of viscosity is defined as the viscous force acting per unit area between two layers moving with unit velocity gradient.

(1) Units : dyne-s-cm^{-2} or Poise (C.G.S. system); Newton-s-m^{-2} or Poiseuille or decapoise (S.I. system)

1 Poiseuille = 1 decapoise = 10 Poise

(2) Dimension : $[ML^{-1}T^{-1}]$

(3) Viscosity of liquid is much greater (about 100 times more) than that of gases i.e. $\eta_L > \eta_G$

Example : Viscosity of water = 0.01 Poise while of air = 200 μ Poise

(4) With increase in pressure, the viscosity of liquids (except water) increases while that of gases is practically independent of pressure. The viscosity of water decreases with increase in pressure.

(5) **Difference between viscosity and solid friction :** Viscosity differs from the solid friction in the respect that the viscous force acting between two layers of the liquid depends upon the area of the layers, the relative velocity of two layers and distance between two layers, but the friction between two solid surfaces is independent of the area of surfaces in contact and the relative velocity between them.

(6) From kinetic theory point of view viscosity represents transport of momentum, while diffusion and conduction represents transport of mass and energy respectively.

(7) The viscosity of thick liquids like honey, glycerin, coaltar *etc*. is more than that of thin liquids like water.

(8) The cause of viscosity in liquids is cohesive forces among molecules where as in gases it is due to diffusion.

(9) The viscosity of gases increases with increase of temperature, because on increasing temperature the rate of diffusion increases.

(10) The viscosity of liquid decreases with increase of temperature, because the cohesive force between the liquid molecules decreases with increase of temperature

Relation between coefficient of viscosity and temperature; Andrade formula $\eta = \frac{A e^{C\rho/T}}{\rho^{-1/3}}$

Where T= Absolute temperature of liquid, ρ = density of liquid, A and C are constants.

Sample problems based on Viscosity

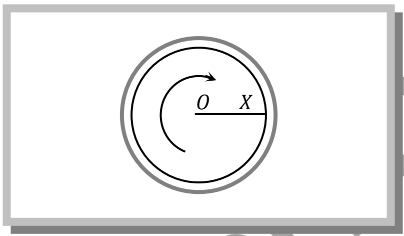
Problem 48. A square plate of 0.1 *m* side moves parallel to a second plate with a velocity of 0.1 *m/s*, both plates being immersed in water. If the viscous force is 0.002 *N* and the coefficient of viscosity is 0.01 *poise*, distance between the plates in *m* is

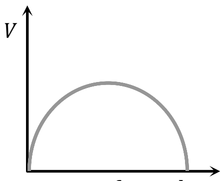
(a) 0.1 (b) 0.05 (c) 0.005 (d) 0.0005

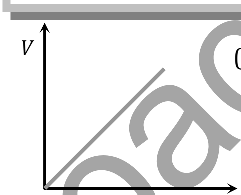
Solution : (d) $A = (0.1)^2 = 0.01 m^2$, $\eta = 0.01$ $Poise = 0.001$ $decapoise$ (M.K.S. unit), dv = 0.1 *m/s* and F = 0.002 *N*

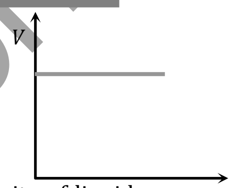
$$F = \eta A \frac{dv}{dx} \quad \therefore \; dx = \frac{\eta A dv}{F} = \frac{0.001 \times 0.01 \times 0.1}{0.002} = 0.0005 \; m.$$

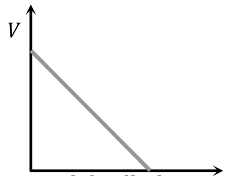
Problem 49. The diagram shows a cup of tea seen from above. The tea has been stirred and is now rotating without turbulence. A graph showing the speed *v* with which the liquid is crossing points at a distance *X* from *O* along a radius *XO* would look like



(a)


(b)


(c)


(d)


Solution : (d) When we move from the centre to the circumference the velocity of liquid goes on decreasing and finally becomes zero

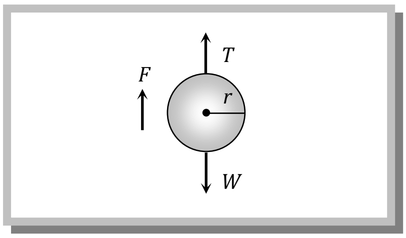
11.15 Stoke's Law and Terminal Velocity

When a body moves through a fluid, the fluid in contact with the body is dragged with it. This establishes relative motion in fluid layers near the body, due to which viscous force starts operating. The fluid exerts viscous force on the body to oppose its motion. The magnitude of the viscous force depends on the shape and size of the body, its speed and the viscosity of the fluid. Stokes established that if a sphere of radius *r* moves with velocity *v* through a fluid of viscosity η, the viscous force opposing the motion of the sphere is

$F = 6\pi\eta r v$

This law is called Stokes law.

If a spherical body of radius *r* is dropped in a viscous fluid, it is first accelerated and then it's acceleration becomes zero and it attains a constant velocity called terminal velocity.



Force on the body

(i) Weight of the body (W) = mg = (*volume* × *density*) × $g = \frac{4}{3}\pi r^3 \rho g$

(ii) Upward thrust (T) = weight of the fluid displaced

= (*volume* × *density*) of the fluid × $g = \frac{4}{3}\pi r^3 \sigma g$

(iii) Viscous force (F) = $6\pi\eta r v$

When the body attains terminal velocity the net force acting on the body is zero. $\therefore$ $W - T - F = 0$ or $F = W - T$

$$\Rightarrow \; 6\pi\eta r v = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g = \frac{4}{3}\pi r^3 (\rho - \sigma) g$$

∴ Terminal velocity $v = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$

(i) Terminal velocity depend on the radius of the sphere so if radius is made n - fold, terminal velocity will become n^2 times.

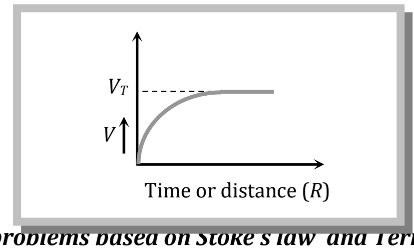
(ii) Greater the density of solid greater will be the terminal velocity

(iii) Greater the density and viscosity of the fluid lesser will be the terminal velocity.

(iv) If $\rho > \sigma$ then terminal velocity will be positive and hence the spherical body will attain constant velocity in downward direction.

(v) If $\rho < \sigma$ then terminal velocity will be negative and hence the spherical body will attain constant velocity in upward direction. Example : Air bubble in a liquid and clouds in sky.

(vi) Terminal velocity graph :



Sample problems based on Stoke's law and Terminal velocity

Problem **50.** Spherical balls of radius 'r' are falling in a viscous fluid of viscosity 'η' with a velocity 'v'. The retarding viscous force acting on the spherical ball is **[AIEEE 2004]**

(a) Inversely proportional to 'r' but directly proportional to velocity 'v'

(b) Directly proportional to both radius 'r' and velocity 'v'

(c) Inversely proportional to both radius 'r' and velocity 'v'

(d) Directly proportional to 'r' but inversely proportional to 'v'

Solution : (b) $F = 6\pi\eta rv$

Problem **51.** A small sphere of mass m is dropped from a great height. After it has fallen 100 m, it has attained its terminal velocity and continues to fall at that speed. The work done by air friction against the sphere during the first 100 m of fall is

(a) Greater than the work done by air friction in the second 100 m

(b) Less than the work done by air friction in the second 100 m

(c) Equal to 100 mg

(d) Greater than 100 mg

Solution : (b) In the first 100 m body starts from rest and its velocity goes on increasing and after 100 m it acquire maximum velocity (terminal velocity). Further, air friction *i.e.* viscous force which is proportional to velocity is low in the beginning and maximum at $v = v_T$.

Hence work done against air friction in the first 100 m is less than the work done in next 100 m.

Problem **52.** Two drops of the same radius are falling through air with a steady velocity of 5 cm per sec. If the two drops coalesce, the terminal velocity would be **[MP PMT 1990]**

(a) 10 cm per sec (b) 2.5 cm per sec (c) $5 \times (4)^{1/3}$ cm per sec (d) $5 \times \sqrt{2}$ cm per sec

Solution : (c) If two drops of same radius r coalesce then radius of new drop is given by R

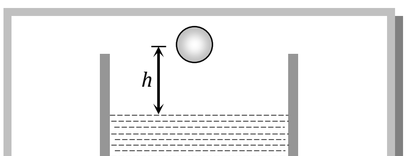
$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r^3 + \frac{4}{3}\pi r^3 \Rightarrow R^3 = 2r^3 \Rightarrow R = 2^{1/3} r$$

If drop of radius r is falling in viscous medium then it acquire a critical velocity v and $v \propto r^2$

$$\frac{v_2}{v_1} = \left(\frac{R}{r}\right)^2 = \left(\frac{2^{1/3} r}{r}\right)^2 \Rightarrow v_2 = 2^{2/3} \times v_1 = 2^{2/3} \times (5) = 5 \times (4)^{1/3} m/s$$

Problem **53.** A ball of radius r and density ρ falls freely under gravity through a distance h before entering water. Velocity of ball does not change even on entering water. If viscosity of water is η, the value of h is given by

(a) $\frac{2}{9} r^2 \left(\frac{1-\rho}{\eta}\right) g$



(b) $\dfrac{2}{81} r^2 \left(\dfrac{\rho - 1}{\eta}\right) g$

(c) $\dfrac{2}{81} r^4 \left(\dfrac{\rho - 1}{\eta}\right)^2 g$

(d) $\dfrac{2}{9} r^4 \left(\dfrac{\rho - 1}{\eta}\right)^2 g$

Solution : (c) Velocity of ball when it strikes the water surface $v = \sqrt{2gh}$ ....(i)

Terminal velocity of ball inside the water $v = \dfrac{2}{9} r^2 g \dfrac{(\rho - 1)}{\eta}$ ....(ii)

Equating (i) and (ii) we get $\sqrt{2gh} = \dfrac{2}{9} \dfrac{r^2 g}{\eta} (\rho - 1) \Rightarrow h = \dfrac{2}{81} r^4 \left(\dfrac{\rho - 1}{\eta}\right)^2 g$

11.16 Poiseuille's Formula

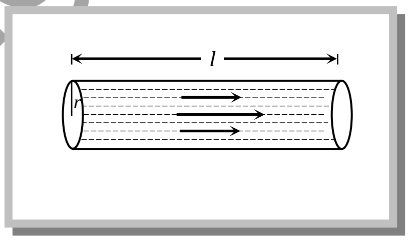
Poiseuille studied the stream-line flow of liquid in capillary tubes. He found that if a pressure difference (P) is maintained across the two ends of a capillary tube of length 'l' and radius r, then the volume of liquid coming out of the tube per second is

(i) Directly proportional to the pressure difference (P).

(ii) Directly proportional to the fourth power of radius (r) of the capillary tube

(iii) Inversely proportional to the coefficient of viscosity (η) of the liquid.

(iv) Inversely proportional to the length (l) of the capillary tube.



i.e. $V \propto \dfrac{P r^4}{\eta l}$ or $V = \dfrac{KP r^4}{\eta l}$

$\therefore$ $V = \dfrac{\pi P r^4}{8 \eta l}$ [Where $K = \dfrac{\pi}{8}$ is the constant of proportionality]

This is known as Poiseulle's equation.

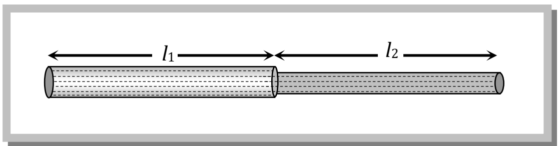
This equation also can be written as, $V = \dfrac{P}{R}$ where $R = \dfrac{8 \eta l}{\pi r^4}$

R is called as liquid resistance.

(1) **Series combination of tubes**

(i) When two tubes of length l_1 , l_2 and radii r_1 r_2 are connected in series across a pressure difference P,

Then $P = P_1 + P_2$ ......(i)



Where P_1 and P_2 are the pressure difference across the first and second tube respectively

(ii) The volume of liquid flowing through both the tubes *i.e.* rate of flow of liquid is same.

Therefore $V = V_1 = V_2$

i.e., $V = \dfrac{\pi P_1 r_1^4}{8 \eta l_1} = \dfrac{\pi P_2 r_2^4}{8 \eta l_2}$ .....(ii)

Substituting the value of P_1 and P_2 from equation (ii) to equation (i) we get

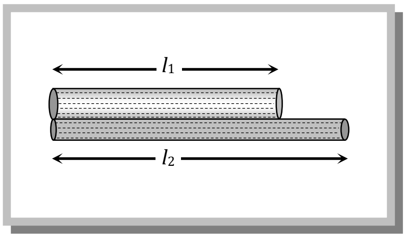
$$P = P_1 + P_2 = V\left[\frac{8\eta l_1}{\pi r_1^4} + \frac{8\eta l_2}{\pi r_2^4}\right]$$

$\therefore$ $V = \dfrac{P}{\left[\dfrac{8\eta l_1}{\pi r_1^4} + \dfrac{8\eta l_2}{\pi r_2^4}\right]} = \dfrac{P}{R_1 + R_2} = \dfrac{P}{R_{eff}}$ Where R_1 and R_2 are the liquid resistance in tubes

(iii) Effective liquid resistance in series combination $R_{eff} = R_1 + R_2$

(2) **Parallel combination of tubes**

(i) $P = P_1 = P_2$



(ii) $V = V_1 + V_2 = \frac{P\pi r_1^4}{8\eta l_1} + \frac{P\pi r_2^4}{8\eta l_2} = P\left[\frac{\pi r_1^4}{8\eta l_1} + \frac{\pi r_2^4}{8\eta l_2}\right]$

$\therefore \; V = P\left[\frac{1}{R_1} + \frac{1}{R_2}\right] = \frac{P}{R_{eff}}$

(iii) Effective liquid resistance in parallel combination

$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2}$ or $R_{eff} = \frac{R_1 R_2}{R_1 + R_2}$

Sample problems based on Poiseuille's law

<u>*Problem*</u> 54. The rate of steady volume flow of water through a capillary tube of length 'l' and radius 'r' under a pressure difference of P is V. This tube is connected with another tube of the same length but half the radius in series. Then the rate of steady volume flow through them is (The pressure difference across the combination is P)

[EAMCET (Engg.) 2003]

(a) $\frac{V}{16}$ (b) $\frac{V}{17}$ (c) $\frac{16V}{17}$ (d) $\frac{17V}{16}$

Solution : (b) Rate of flow of liquid $V = \frac{P}{R}$ where liquid resistance $R = \frac{8\eta l}{\pi r^4}$

For another tube liquid resistance $R' = \frac{8\eta l}{\pi\left(\frac{r}{2}\right)^4} = \frac{8\eta l}{\pi r^4}.16 = 16R$

For the series combination $V_{New} = \frac{P}{R + R'} = \frac{P}{R + 16R} = \frac{P}{17R} = \frac{V}{17}$.

<u>*Problem*</u> 55. A liquid is flowing in a horizontal uniform capillary tube under a constant pressure difference P. The value of pressure for which the rate of flow of the liquid is doubled when the radius and length both are doubled is

[EAMCET 2001]

(a) P (b) $\frac{3P}{4}$ (c) $\frac{P}{2}$ (d) $\frac{P}{4}$

Solution : (d) From $V = \frac{P\pi r^4}{8\eta l} \Rightarrow P = \frac{V8\eta l}{\pi r^4} \Rightarrow \frac{P_2}{P_1} = \frac{V_2}{V_1} \times \frac{l_2}{l_1} \times \left(\frac{r_1}{r_2}\right)^4 = 2 \times 2 \times \left(\frac{1}{2}\right)^4 = \frac{1}{4} \Rightarrow P_2 = \frac{P_1}{4} = \frac{P}{4}$.

<u>*Problem*</u> 56. Two capillary tubes of same radius r but of lengths l_1 and l_2 are fitted in parallel to the bottom of a vessel. The pressure head is P. What should be the length of a single tube that can replace the two tubes so that the rate of flow is same as before

(a) $l_1 + l_2$ (b) $\frac{1}{l_1} + \frac{1}{l_2}$ (c) $\frac{l_1 l_2}{l_1 + l_2}$ (d) $\frac{1}{l_1 + l_2}$

Solution : (c) For parallel combination $\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{\pi r^4}{8\eta l} = \frac{\pi r^4}{8\eta l_1} + \frac{\pi r^4}{8\eta l_2} \Rightarrow \frac{1}{l} = \frac{1}{l_1} + \frac{1}{l_2}$ $\therefore l = \frac{l_1 l_2}{l_1 + l_2}$

<u>*Problem*</u> 57. We have two (narrow) capillary tubes T_1 and T_2. Their lengths are l_1 and l_2 and radii of cross-section are r_1 and r_2 respectively. The rate of flow of water under a pressure difference P through tube T_1 is $8 cm^3/sec$. If $l_1 = 2l_2$ and $r_1 = r_2$, what will be the rate of flow when the two tubes are connected in series and pressure difference across the combination is same as before (= P)

(a) $4\ cm^3/sec$ (b) $(16/3)\ cm^3/sec$ (c) $(8/17)\ cm^3/sec$ (d) None of these

Solution : (b) $V = \frac{\pi \Pr^4}{8\eta l} = \frac{8 cm^3}{\sec}$

For composite tube $V_1 = \frac{P\pi r^4}{8\eta\left(l + \frac{l}{2}\right)} = \frac{2}{3}\frac{\pi P r^4}{8\eta l} = \frac{2}{3} \times 8 = \frac{16}{3}\frac{cm^3}{\sec}$ $\left[\because\ l_1 = l = 2l_2 \text{ or } l_2 = \frac{l}{2}\right]$

***Problem* 58.** A capillary tube is attached horizontally to a constant head arrangement. If the radius of the capillary tube is increased by 10% then the rate of flow of liquid will change nearly by

(a) + 10% (b) + 46% (c) – 10% (d) – 40%

Solution : (b) $V = \frac{P\pi r^4}{8\eta l} \Rightarrow \frac{V_2}{V_1} = \left(\frac{r_2}{r_1}\right)^4 \Rightarrow V_2 = V_1\left(\frac{110}{100}\right)^4 = V_1 (1.1)^4 = 1.4641 V$

$\frac{\Delta V}{V} = \frac{V_2 - V_1}{V} = \frac{1.4641\ V - V}{V} = 0.46$ or 46%.